Do not write in these boxes:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ

Instructor:____________________ Name: Solutions

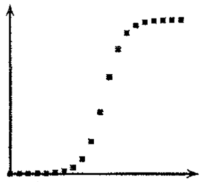
NORTHEASTERN UNIVERSITY

Department of Mathematics

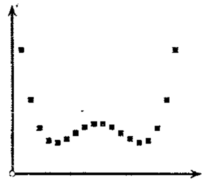
MTH U131 (Calculus for Business and Economics) **Final Exam** **Fall 2007**

Instructions: Put your name and your instructor's name in the blanks above. Put your final answers to each question in the designated spaces on these test pages. (You may lose all credit for a problem if you don't.) **SHOW YOUR WORK.** If there is not enough room to show your work, use the back of the preceding page. **Whenever you use nDeriv or fnInt on your calculator, say so.** Always tell what function is in Y_1, Y_2, etc. in your calculator. For your convenience, there is a table of formulas at the end of the exam. On this exam, you may only use one of the following calculators: **TI-83, TI-83$^+$, TI-84 or TI-84+.** **Total Points = 100**

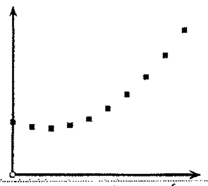
1. For each scatter plot below, tell whether it is best described as: Linear, Exponential, Quadratic, Cubic, Logistic or None. The same answer might be used more than once. (1 point each)

a. 

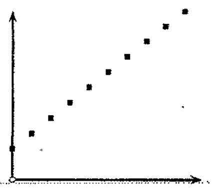
logistic

b. 

none

c. 

quadratic

d. 

linear

2. The number of crimes committed in the U.S., in millions , t years after 1979 is given in the following table.

t, years after 1979	2	4	6	8	10	12
Number of crimes, in millions.	13.4	12.1	12.4	13.5	14.2	14.8

a) (3 points) Let $N(t)$ be the number of millions of crimes committed in the U.S., t years after 1979. Use the table above to fit the best model for $N(t)$, **among the following choices: LINEAR, QUADRATIC, CUBIC, EXPONENTIAL and LOGISTIC**. Write the formula for the model for $N(t)$ here rounding each coefficient to 3 decimal places.

$$N(t) = -.014t^3 + .344t^2 - 2.264t + 16.633$$

b) (2 points) Use the FULL (i.e, EXACT model) for $N(t)$ to approximate the average rate of change of the number of crimes committed in the U.S. from 1984 to 1990. Show work and give your answer (rounded to two decimal places) with units.

Y_1 = full model, $\dfrac{Y_1(11) - Y_1(5)}{11-5} = .42$ million crimes per year

3. Find the derivative of each of the following functions (5 points each).

a) $m(x) = \ln(3x + 10^{-x}) - \frac{7.25}{x^6}$ $= \ln(3x+10^{-x}) - 7.25x^{-6}$

$$m'(x) = \left(\frac{1}{3x+10^{-x}}\right) \cdot (3x+10^{-x})' - 7.25(-6)x^{-7}$$

$$= \frac{3 - 10^{-x}\ln(10)}{3x+10^{-x}} + 43.5x^{-7}$$

b) $g(x) = 3(1.46)^x - 6\sqrt{x^9} + 5x^{-3} - \frac{\ln(x)}{\ln(2)}$ $= 3(1.46)^x - 6x^{9/2} + 5x^{-3} - \frac{\ln(x)}{\ln(2)}$

$$g'(x) = 3(1.46)^x\ln(1.46) - 6\left(\tfrac{9}{2}\right)x^{7/2} + 5(-3)x^{-4} - \frac{1}{\ln(2)} \cdot \frac{1}{x}$$

or

$$3(1.46)^x\ln(1.46) - 27x^{7/2} - 15x^{-4} - \frac{1}{(\ln 2)x}$$

c) $h(x) = (10\sqrt[3]{x})(3^{2x} - x^2)$ $= 10x^{1/3}(3^{2x} - x^2)$

$$h'(x) = 10x^{1/3}(3^{2x}-x^2)' + (10x^{1/3})' \cdot (3^{2x}-x^2)$$

$$= 10x^{1/3}(3^{2x}\,2\ln 3 - 2x) + \left(\tfrac{1}{3}\right)10x^{-2/3}(3^{2x}-x^2)$$

d) $f(x) = \frac{90}{1+5e^{-0.04x}} - 8e^x$ $= 90(1+5e^{-.04x})^{-1} - 8e^x$

$$f' = 90(-1)(1+5e^{-.04x})^{-2}(1+5e^{-.04x})' - 8e^x$$

$$= -90(1+5e^{-.04x})^{-2}(5(-.04)e^{-.04x}) - 8e^x$$

or

$$= 18e^{-.04x}(1+5e^{-.04x})^{-2} - 8e^x$$

4. Let $F(x)$ be the number of DVRs sold, in thousands, if the price of each DVR is x hundred dollars. In parts a), b), and c), of this problem circle the number of the correct answer. **There is only one correct answer and there is no partial credit.**

a) (2 points) The practical meaning of $F(3) = 2$ is:

(i) When the price is \$200, 3 thousand DVRs are sold.

(ii) When the price is \$300, 2 thousand DVRs are sold.

(iii) When the price is \$3000, 200 DVRs are sold.

(iv) When the price is \$2000, 300 DVRs are sold.

(v) None of the above

b) (2 points) Give the units of $F'(x)$:

(i) dollars per DVR

(ii) hundreds of DVRs per thousand dollars

(iii) thousands of DVRs per dollar

(iv) DVRs per dollar

(v) None of the above

c) (2 points) If $F(3) = 2$ and $F'(3) = -0.4$, give the best estimate for the number of DVRs that will be sold if the price of a DVR is \$325.

(i) 1975 (ii) 2100 (iii) 1900 (iv) 1600 (v) 2025

5. A chain of home and garden supply stores sells holiday wreaths.

a) (2 points) The chain's revenue (in dollars) from selling wreaths is

$$R(x) = 12x^2 + 60x + 50,$$

where x is the number of hundreds of wreaths sold. Write the formula for the **MARGINAL REVENUE** function. Include the unit in your answer.

$R'(x) = 24x + 60$ dollars per hundred wreaths

b) (2 points) Suppose that **MARGINAL REVENUE** of the chain in part (a) from selling 6000 wreaths is \$15 per wreath. The practical meaning of this fact is: (Circle the number of the correct answer. There is only one, and there is no partial credit.)

(i) The revenue from selling the 6001^{st} wreath is about \$15.

(ii) The revenue from selling 6000 wreaths is exactly \$90,000.

(iii) The revenue from selling 6000 wreaths is about \$90,000.

(iv) The average revenue from selling one wreath is about \$15.

(v) None of the above

6. An online computer store's profit function for selling 2GB digital memory cards is

$$P(x) = (25x^2 + 50x - 250)(0.912)^x \ ,$$

where $P(x)$ is the profit in **dollars** when the selling price of each memory card is x **dollars**. Carefully enter the function $P(x)$ into Y_1 on your calculator. **Check that you have entered the function correctly by computing $P(3)$. You should obtain 94.818816.**

a) (5 points) From past sales experience, the owner of the store knows that she shouldn't sell a 2 GB digital memory card for more than \$30. Use *nDeriv* and your calculator to determine the price that maximizes PROFIT. Write the answer with all the decimal places the calculator gives.

calculator answer: 21.207256

Circle the equation you solved:

(i) $nDeriv(P(x), x, x) = P(x)$ (ii) $nDeriv(P(x), x, x) = 0$ (iii) $nDeriv(P''(x), x, x) = 0$

(iv) $nDeriv(P'(x), x, x) = 0$ (v) $nDeriv(P'(x), x, x) = P(x)$

Circle the calculator procedure you used:

(i) *2nd/Calc/Zero* (ii) *2nd/Calc/Intersect* (iii) *SOLVER*

b) (3 points) By comparing function values, show that the price you found in part (a) maximizes the profit.

$P(21) = 1708.805977 < P(21.207256) = 1708.96623$

$P(22) = 1706.696161 < P(21.207256) = 1708.96623$

c) (3 points) Use the second derivative test to show that the price you found in part (a) maximizes the profit.

$Y_1 = P(x)$, $Y_2 = nDeriv(Y_1, x, x)$, $Y_3 = nDeriv(Y_2, x, x)$

$Y_3(21.207256) = -7.4122 < 0$

7. (3 points) *Monica's Sweets* sells boxes of assorted chocolates. Based on a marketing poll, Monica has found the following model for the demand function (i.e., the number of boxes of chocolates sold each week):

$$D(x) = 6 + 200e^{-.04x} \ ,$$

where the price of each box of chocolates is x dollars. Each box of chocolates costs the shop \$9.50 to make. The shop has fixed costs of \$70 for the advertising display for the chocolates. Write down the formulas for $R(x)$, $C(x)$, and $P(x)$, the revenue, cost, and profit functions (in dollars). **Write each formula out in full. Do not use any abbreviations.**

R(x) = $x(6+200e^{-.04x})$

C(x) = $9.5(6+200e^{-.04x}) + 70$

P(x) = $x(6+200e^{-.04x}) - [9.5(6+200e^{-.04x}) + 70]$

8. Read this problem carefully. **Parts (a) and (b) of this problem are about DIFFERENT functions.**

a) (9 points) Suppose that $f'(x) = (4x - 11)(x + 2)$, i.e., the DERIVATIVE of $f(x)$ is $(4x - 11)(x + 2)$.

(i) Find the x-coordinates of the critical points of $f(x)$. USE ALGEBRA. Show your work especially the equation you must solve and how you solve it.

$(4x - 11)(x + 2) = 0$

$4x - 11 = 0 \qquad x + 2 = 0$

$4x = 11, \quad x = \frac{11}{4} \qquad x = -2$

$x = -2, \quad x = \frac{11}{4} = 2.75$

(ii) Find the x-coordinate of a relative maximum of $f(x)$. SHOW how you know it is a relative maximum.

$f'(-3) = 23 > 0, \quad f'(-1) = -15 < 0$

So $x = -2$ is a relative maximum by the first derivative test

$x = -2$

(iii) Find the x-coordinate of a relative minimum of $f(x)$. SHOW how you know it is a relative minimum.

$f'(2) = -12 < 0, \quad f'(3) = 5 > 0$

So $x = \frac{11}{4}$ is a relative minimum by the first derivative test

$x = \frac{11}{4}$

b) (5 points) (i) **Use derivatives and algebra** to find the inflection point of the function $h(x) = 2x^3 - 9x^2 - 24x + 80$. Give its x and y coordinates. Show your work especially the equation you must solve and how you solve it.

$h'(x) = 6x^2 - 18x - 24$

$h''(x) = 12x - 18$

$12x - 18 = 0 \Rightarrow 12x = 18, \quad x = \frac{18}{12} = 1.5$

$h(1.5) = 30.5$

inflection point has coordinates $(1.5, 30.5)$

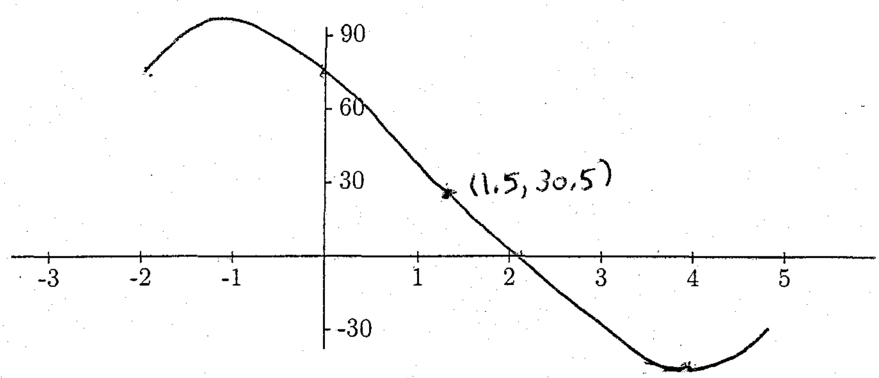
(ii) Graph $h(x)$ (from part (i)) using the window: $X_{min} = -2$, $X_{max} = 5$, $Y_{min} = -40$, $Y_{max} = 100$. Sketch what you see below. In your sketch, **label the inflection point clearly with its coordinates.** Use the graph to decide what kind of inflection point the graph has. Circle one of the following.

Point of fastest increase

Point of slowest increase

Point of fastest decrease

Point of slowest decrease



9. (3 points) Evaluate the integral, i.e., find the general antiderivative.

$$\int (4.5e^x - 17x^7 - \frac{\ln(10)}{5})\, dx$$

$$= 4.5e^x - \frac{17}{8}x^8 - \frac{\ln(10)}{5}x + C$$

10. In parts a), b), c), and d) circle the number of the correct answer. (2 points each, no partial credit).

a) An antiderivative of $k(x) = \frac{3}{2x^6}$ is:

(i) $\frac{21}{2x^7}$ (ii) $-\frac{6}{5}x^{-5}$ (iii) $\frac{3}{2}\ln(|x^6|)$ (iv) $-\frac{3}{10}x^{-5}$ (v) None of the above

b) An antiderivative of $m(x) = \frac{d(xe^x)}{dx}$ is:

(i) $\frac{x^2e^x}{2}$ (ii) xe^x (iii) $xe^x + x$ (iv) $\frac{e^{x^2}}{2}$ (v) None of the above

c) An antiderivative of $g(x) = \frac{1}{6\sqrt{x}}$ is:

(i) $\ln(6\sqrt{x})$ (ii) $\frac{x^{1/2}}{6}$ (iii) $\frac{\ln(|x^{1/2}|)}{6}$ (iv) $\frac{1}{4x^{3/2}}$ (v) None of the above

d) An antiderivative of $h(x) = 5(1.2)^x$ is:

(i) $5(1.2)^x \ln(1.2)$ (ii) $\frac{6^x}{\ln(6)}$ (iii) $\frac{5(1.2)^x}{\ln(1.2)}$ (iv) $6^x \ln(6)$ (v) None of the above

11. (5 points) Evaluate the following integral. Show all work. *Use your calculator only to perform basic arithmetic.* Do not use *fnInt*. **A numerical answer without appropriate work shown will receive no credit.**

$$\int_1^9 (81x^{-3} - \frac{x^{3/2}}{10})\, dx$$

$$= \left[\frac{81x^{-2}}{-2} - \frac{x^{5/2}}{10 \cdot 5/2}\right]_1^9 = \left(\frac{81(9^{-2})}{-2} - \frac{9^{5/2}}{25}\right) - \left(\frac{81}{-2} - \frac{1}{25}\right)$$

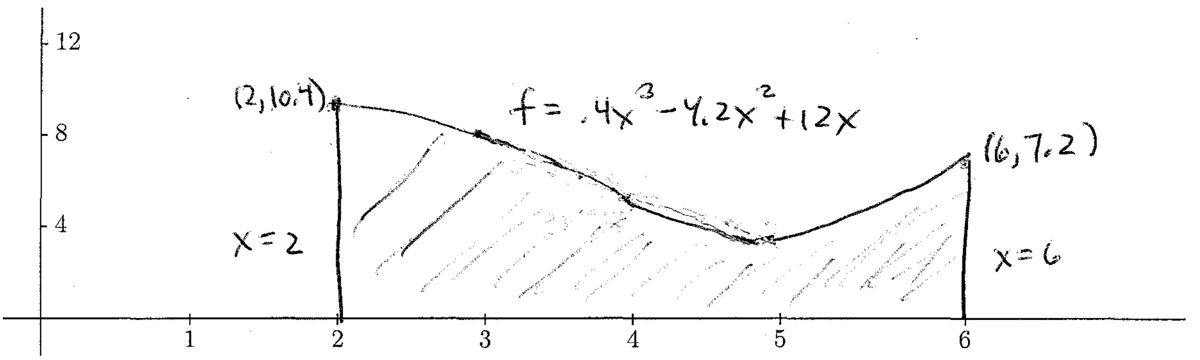
$$= -10.22 - (-40.54)$$

$$= 30.32$$

12a. (2 points) Make a careful sketch of the region whose area is given by the definite integral:

$$\int_2^6 (0.4x^3 - 4.2x^2 + 12x)\, dx.$$

Label the boundary curves (with their equations) and corner points (with their coordinates) of the region. Then shade the region.



12b. (2 points) The 4 left rectangle approximation (in square units) of the area in part (a) is: (Circle the number of the correct answer). **There is no partial credit.**

(i) 33.6 (ii) 30.8 (iii) 29.6 (iv) 27.2 (v) None of the above

No work need be shown

x	f(x)	area of rectangle
2	10.4	$(10.4)\cdot 1 = 10.4$
3	9	$(9)\cdot 1 = 9$
4	6.4	$(6.4)\cdot 1 = 6.4$
5	5	$(5)\cdot 1 = 5$

4 left rectangle approximation

$= 10.4 + 9 + 6.4 + 5$

$= 30.8$ square units

13. (4 points) Find $F(x)$, the specific antiderivative of the function $f(x)$, where $f(x) = 96x^2 - 1024x^4$ and $F(1/4) = 5$. Show all your work especially **any equations you solve.**

$$F(x) = \int f(x)dx = \int (96x^2 - 1024x^4)dx$$

$$= \frac{96x^3}{3} - \frac{1024x^5}{5} + C \quad \text{or} \quad 32x^2 - 204.8x^5 + C$$

$$5 = F(\tfrac{1}{4}) = \frac{96}{3}(\tfrac{1}{4})^3 - \frac{1024}{5}(\tfrac{1}{4})^5 + C$$

$$5 = 0.5 - 0.2 + C$$

$$C = 5 - 0.5 + .2 = 4.7 \qquad F = 32x^3 - 204.8x^5 + 4.7$$

14. The balance (in dollars) in a savings account t years after January 1, 2002 is given by the formula:

$$A(t) = 5000(1.04)^t.$$

a) (3 points) Find the rate of change of the balance in the account on January 1, 2007. Show work. Give your answer (rounded to two decimal places) with units.

$A'(t) = 5000(1.04)^t \ln(1.04)$

$A'(5) = 238.59$ dollars per year

or nDeriv$(A(x), x, 5) = 238.59$ dollars per year

b) (3 points) Write a formula involving a definite integral for the average balance in the account between January 1, 2003 and January 1, 2007.

average balance $= \dfrac{1}{5-1} \displaystyle\int_1^5 5000(1.04)^t \, dt$

Circle the correct answer for the average balance (rounded to two decimal places) in the account between January 1, 2003 and January 1, 2007.

(i) \$5966.28 (ii)\$6083.26 (iii)\$5630.09 (iv)\$5541.63 (v) None of the above

No work need be shown

average balance $= \frac{1}{4} \displaystyle\int_1^5 5000(1.04)^t dt \approx \frac{1}{4}$ fnInt$(5000(1.04)^x, x, 1, 5) = \$5630.09$

15. (3 points) Let $p(t)$ be the rate of change of the U.S. consumer credit card debt, in billions of dollars per year, t years after 1995. What is the practical meaning of the following integral?

$$\int_3^8 p(t)\, dt$$

Circle the number of the correct answer. (There is only one, and there is no partial credit.)

(i) The average U.S. consumer credit card debt in billions of dollars from 1998 to 2003.

(ii) The change in the rate of change of the U.S. consumer credit card debt from 1998 and 2003 in billions of dollars per year

(iii) The change in the U.S. consumer credit card debt from 1998 to 2003 in billions of dollars

(iv) The average rate of change of the U.S. consumer credit card debt from 1998 to 2003 in billions of dollars per year

(v) None of the above