

Abstract

The problem of tensor product multiplicities is an old problem in representation theory of simple Lie algebras, which asks how many times certain irreducible finite dimensional module appears in tensor product of two such modules. In 1983, Gelfand and Zelevinsky initiated study of “canonical basis” as a tool to attack this problem. For longest time the term “canonical basis” remained intuitive, and was given firm mathematical meaning only recently by George Lusztig and Masaki Kashiwara.

The theory of cluster algebras, recently formed and developed by Fomin and Zelevinsky, is aimed at (besides other things) describing canonical basis in concrete combinatorial terms. The coordinate rings of various interesting varieties appearing in representation theory carry a structure of cluster algebras.

In this talk I will give definition of a cluster algebra (of geometric type) motivated by combinatorics of “canonical basis”. I will use simple Lie groups of rank 2 as guiding examples. For the case of G_2 , the picture is still incomplete. Time permit, I will explain a recent conjecture of Geiss, Leclerc and Schröer and a proof of this conjecture for the case of G_2 .