Polyhedra, Complexes, and Symmetry

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Symmetric polyhedra have been investigated since antiquity. With the passage of time, the concept of a polyhedron has undergone a number of changes which have brought to light new classes of highly-symmetric polyhedra. Coxeter's famous "Regular Polytopes" and his various other writings treat the Platonic solids, the Kepler-Poinsot polyhedra and the Petrie-Coxeter polyhedra in great detail, and cover what might be called the classical theory.

A lot has happened in this area in the past 30 years. Around 1975, Grünbaum initiated the skeletal approach to polyhedra and symmetry, which is essentially graph-theoretical. A polyhedron is viewed as a geometric (edge) graph in space equipped with additional structure imposed by the faces, and its symmetry is measured by transitivity properties of its geometric symmetry group.

We describe the regular or chiral polyhedra in Euclidean 3-space. Regular, or reflexibly regular, polyhedra have a geometric symmetry group which is transitive on the flags. Chiral, or irreflexibly regular, polyhedra are nearly regular polyhedra; their geometric symmetry groups have two orbits on the flags such that adjacent flags are in distinct orbits. The geometry and combinatorics of these polyhedra is generally quite complicated. There are 48 regular polyhedra but several infinite families of chiral polyhedra.

We also present work in progress on the full classification of regular polygonal complexes in 3-space. Polygonal complexes are more general than polyhedra, in that they usually have more than two faces meeting at an edge, but otherwise share many of their properties.