Some open problems in geometry of equivariant sets.

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Abstract.

It is well known that the set $Hom_K(K^n, K^m)$ of $m \times n$ matrices with the action of the product of two general linear groups $GL(m, K) \times GL(n, K)$ by row and column operations has finitely many orbits (given by matrices of a given rank). An alternative description is in terms of tensors: Identifiyng $Hom_K(K^n, K^m)$ with $(K^n)^* \otimes K^m$ we will say that a tensor has rank r if it can be written as a sum of r decomposable tensors of the form $v \otimes w$ ($v \in (K^n)^*$, $w \in K^m$).

I will discuss a more general setting: the representations of reductive groups with finitely many orbits. These representations were classified by Kac, however still not much is known about the orbit closures in such situations.

The analysis of these orbit closures and their singularities is an on-going area of research. I will describe some relevant techniques and open questions.

I will also describe a generalization of the notion of rank to higher dimensional tensors and other representations (not nessesarily with finitely many orbits). The problem of maximal tensor rank in these cases is still unsolved.