

Polyhedral Embeddings and Immersions of Triangulated 2-Manifolds in \mathbb{R}^3

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A polyhedral realization of a triangulated 2-manifold is a simplexwise linear embedding into \mathbb{R}^3 in the orientable case or a simplexwise linear immersion in the non-orientable case. It is an open question what the minimum number of vertices is for a realizable triangulation of a certain 2-manifold. Progress has been made in recent years with the aid of computer programs, but few results are known in the non-orientable case. This presentation will cover work on an adapted computer heuristic to produce polyhedral embeddings and immersions with certain symmetries and few vertices. Additionally, it will focus on topological constraints preventing realizability in the non-orientable case, thus narrowing the bounds from both sides. Combining topological, geometric, combinatorial, and computational tools, new minimality results have been obtained. For example, for the projective plane with two handles 10 vertices suffice, and 9 do not.