Elliptic Operators in Divergence Form:
Regularity Theory and Applications

Elliptic operators in divergence form, i.e. $\mathcal{L}u = \partial_i(a_{ij}(x)\partial_j u)$ plus possible lower-order terms, arise naturally in applications such as electrostatics: $a_{ij}(x) = a_{ji}(x)$ may represent the anisotropic electrical conductivity at a point x of a body Ω, and solving $\mathcal{L}u = 0$ with an appropriate boundary condition on $\partial\Omega$ determines the voltage potential u for the induced electric field. One important theoretical question is regularity: how smooth do the a_{ij} need to be to guarantee that u has the desired smoothness?

Regularity considerations also arise in *inverse problems*, such as determining the electric conductivity only from boundary measurements of the potential. Even in the case that $a_{ij}(x) = \gamma(x)\,\delta_{ij}$, i.e. the conductivity is isotropic, there are inverse problems which are unresolved if γ is not sufficiently smooth.

In this talk, I will describe some recent work with Vladimir Maz'ya concerning regularity theory for elliptic operators in divergence form, as well as some of the open questions associated with inverse problems for isotropic conductivities.