The Lyapunov partial differential equation and applications of Riemannian geometry to stability analysis in control theory

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Abstract

Solving the Lyapunov partial differential equation one can obtain necessary and sufficient conditions for stability of a dynamical system. We establish conditions under which it can be solved, and construct the solution in the case of curve systems — those generated by ∇-Killing vector fields of the connection on the manifold where the system evolves.

The quadratic distance function on a Riemannian manifold can be expressed in terms of the position vector, which in turn can be constructed using geodesic normal coordinates through consideration of the exponential map. The formulas for the derivative of the distance are useful to study Lyapunov stability of dynamical systems, and to build cost functions for optimal control and estimation.