PARTIAL DIFFERENTIAL EQUATIONS 2 (MTH G302)
Professor M.Shubin
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Textbook: *Partial Differential Equations*, by Mikhail Shubin

A preliminary version can be downloaded at

http://mystic.math.neu.edu/shubin/PDE-book-2.pdf

Office: 460 Lake Hall. **Phone:** Ext.5676 E-mail: shubin@neu.edu

Class meetings: Mondays and Wednesdays 7:30 – 9:00 p.m., 544 Nightingale Hall

The course will be independent of the course PDE 1 (G202), though knowledge of basics on classical PDE (Laplace, heat and wave equations) is certainly useful. This knowledge can be also gained from the textbook quoted above.

The course will be dedicated to the theory of distributions, methods of functional analysis in PDE, some topics in spectral theory of partial differential operators and selected topics from non-linear equations. Not all of these topics are covered in the book, but in such cases other sources will be provided.

Distributions (or generalized functions) are a powerful tool in the theory of linear PDE. They provide, for instance, a possibility to differentiate all locally integrable functions infinitely many times (though the result may not be a function). They help developing operational technique to investigate linear PDE with constant coefficients through their fundamental solutions.

Functional analysis was developed in 20th century from an idea to treat functions as points in an infinite-dimensional space. This idea allows a miraculously successful use of rich geometric intuition when dealing with functions. In particular, functional analysis proved to be extremely fruitful in applications to PDE. It also provides an adequate language for quantum mechanics through spectral theory.

The first aim of the course is to provide an introduction to essential results of theory of distributions (including their Fourier transform) with most basic applications to PDE, in particular, elementary potential theory. Then Sobolev spaces of functions with applications to constructing generalized solutions for boundary value problems and to spectral theory will be discussed.

The topics mentioned above will be mostly taken from Chapters 4-11 of the book, though only small parts of Chapters 6 and 9 will be discussed.

Then some elements of quantum mechanics will be explained and its relation with spectral theory will be discussed. Some model examples will be considered.

The remaining time will be spent on selected topics in non-linear PDE.

The grade will be based on homework assignments, though a possibility of topics presentation by students and/or final is not excluded.