Complex Analysis, MTH G204.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Complex Analysis, by Theodore W. Gamelin. Springer-Verlag New York, Inc., 2001. Corrected printing 2003

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Homework assignment no. 4
(due October 7)

A general remark. Problems marked with * have an extended due date, which is not specified now. They are usually more difficult, but on the other hand, they are definitely worth thinking about. Please hand me any one of them when it is ready. Some of them may be topics for projects.

1. Section IV.1, page 106-107: 2(b), 4.
2. Section IV.2, page 109-110: 2.
3. Section IV.3, page 111-113: 1, 2*.
4. Section IV.4, page 116-117: 1(a,b,c,e).
5. Section IV.5, page 119: 1, 2, 5*.
6. Section IV.8, page 128-129: 3, 4(a,b), 8, 10.

Hint to no. 8 in IV.8 (page 129). Do not translate to the formulas for real derivatives, this leads to very long calculations. Instead calculate differentials and use the formula

$$df = \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial \bar{z}} d\bar{z}$$

as the definition of the partial derivatives with respect to z and $\bar{z}$.

8*. Prove that for a complex polynomial $p(z)$ the roots of its derivative $p'(z)$ belong to a convex hull of the roots of p. (The convex hull of a non-empty finite set $S \subset \mathbb{R}^2$ is a minimal convex polygon in $\mathbb{R}^2$ which contains S. It can degenerate to a straight line segment or a point.) In particular, if all roots of p are real, then the same holds for p'. (The latter is easy to deduce from Rolle's theorem of real analysis.)