Geometry 1, MTH G122.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Foundations of Differentiable Manifolds and Lie groups, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

Office: 460 Lake Hall. **Phone:** (617)373-5676 **E-mail:** shubin@neu.edu

Homework assignment no. 11
(due December 7)

1. (a) Prove that there exists a Lie algebra with a basis $\{X, Y, Z\}$ (over $\mathbb{R}$), where X, Y, Z satisfy the following commutation relations:

$$[X, Y] = Z, \quad [X, Z] = [Y, Z] = 0.$$

(b) Find the center of this Lie algebra.

(c) Find a simply connected Lie group corresponding to this Lie algebra.

2. (a) Consider the 2-form

$$\omega = dx_1 \wedge dy_1 + dx_2 \wedge dy_2$$

in $\mathbb{R}^4$ with the coordinates x_1, y_1, x_2, y_2. Prove that the linear transformations of $\mathbb{R}^4$ preserving this form, constitute a Lie group G.

(b) Describe the Lie algebra of G and find its dimension.

(c) Is G compact?