

**Geometry 1, MTH G122.**  
**Fall 2004. Professor Mikhail Shubin.**

**Textbook:**

*Foundations of Differentiable Manifolds and Lie groups*, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

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**Homework assignment no. 2**  
(due September 23)

**1.** Prove equivalence of two definitions of the tangent vector at a point  $p \in M$  (where  $M$  is a manifold):

- (a) an equivalence class of curves passing through  $p$  at the moment  $t = 0$ ;
- (b) differentiation of the ring of germs of  $C^\infty$  functions on  $M$  at  $p$ .

Provide explicitly mutually inverse maps from the set of equivalence classes of curves to the set of differentiations.

How can we see that the set of equivalence classes of curves is a vector space?

**2.** Let  $M$  be a compact manifold of dimension  $d$ , and let  $f : M \rightarrow \mathbf{R}^d$  be a  $C^\infty$  map. Prove that  $f$  can not be everywhere non-singular.