Geometry 1, MTH G122.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Foundations of Differentiable Manifolds and Lie groups, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

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Homework assignment no. 7
(due November 4)

1. Let X be a smooth vector field in $\mathbb{R}^d$, $\omega = dx_1 \wedge dx_2 \wedge \ldots \wedge dx_d$ be the standard volume form.

(a) Calculate $L_X\omega$. (Present the answer in coordinates.)

(b) Find an explicit necessary and sufficient condition (on X) which guarantees that the flow of X preserves ω, i.e. $X_t^*\omega = \omega$, wherever $X_t^*\omega$ is defined.

2. Prove the following formulas for operators acting in differential forms on a manifold M:

(a) $i(X)i(Y) + i(Y)i(X) = 0$;

(b) $L_X i(Y) - i(Y)L_X = i([X, Y])$;

(c) $L_X L_Y - L_Y L_X = L_{[X,Y]}$.

Here X, Y are vector fields on M.