

### Final Exam A - Part 1

1. The message

-71, -44, -4, -5, 30, 18, -71, -45, 38, 21, 7, 4, -51, -31, -77, -48

was encoded using the matrix  $M = \begin{pmatrix} -5 & -3 \\ 3 & 2 \end{pmatrix}$  and the following coding scheme:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
0	1	-1	2	-2	3	-3	4	-4	5	-5	6	-6	7	-7	8	-8	9
S	T	U	V	W	X	Y	Z	blank	\$	,	.	!	?				
-9	10	-10	11	-11	12	-12	13	-13	14	-14	15	-15	16				

Show the complete matrix equation needed to decode this message.

(5 points)

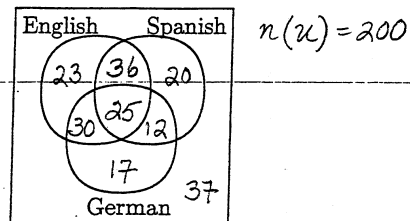
$$\begin{pmatrix} -71 & -44 \\ -4 & -5 \\ 30 & 18 \\ -71 & -45 \\ 38 & 21 \\ 7 & 4 \\ -51 & -31 \\ -77 & -48 \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix} \begin{matrix} 2 \times 2 \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix} = \begin{pmatrix} 10 & -7 \\ -7 & -13 \\ -6 & 0 \\ 7 & -12 \\ -13 & -9 \\ -2 & -1 \\ 9 & -2 \\ 10 & -9 \end{pmatrix}$$

$8 \times 2$

The message is: TOO MANY SECRETS

A phrase used in the movie *Sneakers*

2. Out of a group of 200 applicants for jobs at the World Bank, 114 speak English, 93 speak Spanish, 84 speak German, 61 speak English and Spanish, 37 speak Spanish and German, 55 speak English and German, and 25 speak all three languages. Use this information to *completely* fill in the Venn diagram. Check your arithmetic, there will be no partial credit given. (4 points)





### Final Exam A - Part 1

4. Consider the system of equations  $5x - 4y = -2$   
 $3x - 5y = 5$ .

(a) Express this system as a matrix equation. (1 point)

$$\begin{pmatrix} 5 & -4 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

A B

(b) Use the inverse of the coefficient matrix to solve for  $x$  and  $y$ . Show the new matrix equation you use to do this. (6 points)

$$A^{-1}B = \begin{pmatrix} -30/13 \\ -31/13 \end{pmatrix} = \begin{pmatrix} -2.30769 \\ -2.38462 \end{pmatrix} \quad \left( -\frac{30}{13}, -\frac{31}{13} \right)$$

OR

$$\begin{pmatrix} 5/13 & -4/13 \\ 3/13 & -5/13 \end{pmatrix} \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -30/13 \\ -31/13 \end{pmatrix}$$

$2 \times 2$        $2 \times 1$        $2 \times 1$

5. A state lottery commission separates adult residents into two categories, those who play the lottery regularly and those who do not. Over a two-year period, the probability is .98 that a regular player will continue to play regularly; the probability is .04 that over the same time period a person who does not play regularly will become a regular player.

(a) Construct the transition (stochastic) matrix for this Markov chain. Label each row and column. (4 points)

$$T = \begin{matrix} & \text{PLAY} & \text{DON'T PLAY} \\ \text{PLAY} & \begin{pmatrix} .98 & .04 \end{pmatrix} \\ \text{DON'T PLAY} & \begin{pmatrix} .02 & .96 \end{pmatrix} \end{matrix}_{2 \times 2}$$

(b) Currently, 15% of the population play regularly. Give the initial distribution matrix. (1 point)

$$D_0 = \begin{matrix} \text{PLAY} \\ \text{DON'T PLAY} \end{matrix} \begin{pmatrix} .15 \\ .85 \end{pmatrix}_{2 \times 1}$$

(c) Showing the complete matrix equation, answer the following question. Determine the percentage of the population that will play regularly after 4 years. (4 points)

$$T^4 D_0 = \begin{matrix} \text{PLAY} \\ \text{DON'T PLAY} \end{matrix} \begin{pmatrix} .21014 \\ .78986 \end{pmatrix} \quad \begin{matrix} 21.014\% \text{ WILL PLAY} \\ \text{REGULARLY AFTER 4 YEARS.} \end{matrix}$$

OR

$$T D_0 = \begin{pmatrix} .181 \\ .819 \end{pmatrix} \quad T^2 D_0 = \begin{pmatrix} .21014 \\ .78986 \end{pmatrix}$$

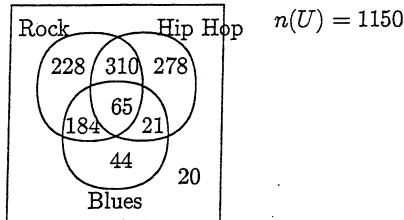
Final Exam A – Part 1

Multiple Choice Section

Put the *letter* of the correct answer in the box at the right side of the page AND fill in the letter on the bubble sheet.

This is the only way to get credit for your answer. **ANSWERS BELOW** ↓

A survey of college students asked them what kind of music they listened to. The information collected is given in the Venn diagram below. Use this diagram to answer questions #2 and #3.



1. **FILL IN THE LETTER 'A' FOR #1 ON THE BUBBLE SHEET.**

Answer # 1: A

2. What is the probability a student in the survey likes blues or hip hop?

- A)  $\frac{343}{1150}$     B)  $\frac{86}{1150}$     C)  $\frac{902}{1150}$     D)  $\frac{837}{1150}$     E)  $\frac{718}{1150}$

Answer # 2: C

3. What is the probability a student in the survey does not like rock?

- A)  $\frac{902}{1150}$     B)  $\frac{922}{1150}$     C)  $\frac{322}{1150}$     D)  $\frac{612}{1150}$     E)  $\frac{363}{1150}$

Answer # 3: E

4. Given that  $n(A \cup B) = 32$ ,  $n(A) = 19$ , and  $n(B) = 25$ , what is  $n(A \cap B)$ ?

- A) 76    B) 12    C) -12    D) 26    E) 10

Answer # 4: B

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$32 = 19 + 25 - x, \quad x = 12$$

5. How many distinct arrangements are there of the letters in the word HONOLULU?

- A) 5040    B) 40320    C) 56    D) 322560    E) 10080

Answer # 5: A

$$\frac{8!}{2!2!2!} = 5040$$

Final Exam A – Part 1 – Multiple Choice

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6. The most popular condiments for hot dogs are mustard, ketchup, onions, chili, sauerkraut, and cheese. How many types of hot dogs can be ordered?

A) 720      B) 46656      C) 64      D) 36      E) 120

Answer # 6:  C

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$$

7. Assuming that repetition is allowed, how many different 4-digit numbers between 5000 and 7000 can be made from the digits 3,4,5,6,7, and 8?

A) 648      B) 16      C) 250      D) 432      E) 375

Answer # 7:  D

$$2 \cdot 6 \cdot 6 \cdot 6 = 432$$

8. An ice cream store stocks 30 flavors and offers a rainbow banana split that contains 3 scoops of ice cream, each of a different flavor. How many different rainbow splits can the store advertise?

A) 24360      B) 812      C) 90      D) 406      E) 4060

Answer # 8:  E

$$C(30, 3) = \frac{30 \cdot 29 \cdot 28}{3 \cdot 2 \cdot 1} = 4060$$

9. A chain with seven stores needs to close five stores, one each month over the next five months. How many different ways can this be done?

A) 42      B) 21      C) 210      D) 2520      E) 16807

Answer # 9:  D

$$P(7, 5) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$$

10. With the Red Sox down 0 to 3 in the Championship Series with the Yankees, a radio announcer said the probability that the Yankees would win was  $\frac{98}{100}$  (in other words, the Red Sox would win when pigs could fly). What did the announcer believe were the odds that the Yankees would beat the Red Sox?

~~A) 98:100      B) 2:100      C) 49:1      D) 1:49      E) 49:50~~

Answer # 10:  C

$$\frac{2}{98} = \frac{1}{49}$$

Final Exam A – Part 1 – Multiple Choice

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The board of directors of a large corporation is made up of 7 women and 9 men. 6 of these members will go as a delegation to a national convention. Use this information to answer questions #11 – #14.

11. How many different delegations are possible?

- A)  $\frac{16!}{7!9!}$       B)  $P(16, 6)$       C)  $C(16, 6)$       D)  $\frac{C(16, 6)}{6!}$       E)  $6!$

Answer # 11:

12. How many of these delegations contain 2 women and 4 men?

- A) 2646      B) 147      C) 127008      D) 3066      E) 1260

Answer # 12:

$$C(7, 2) \cdot C(9, 4) = \frac{7 \cdot 6}{2 \cdot 1} \times \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 2646$$

13. What is the probability that a delegation contains 2 women and 4 men?

- A)  $\frac{90}{1001}$       B)  $\frac{189}{572}$       C)  $\frac{21}{1144}$       D)  $\frac{2646}{5765760}$       E)  $\frac{147}{11440}$

Answer # 13:

$$\frac{\text{Ans to \#12}}{\text{Ans to \#11}} = \frac{2646}{C(16, 6)} = \frac{189}{572}$$

14. What is the probability a delegation contains at least 1 woman?

- A)  $\frac{383}{572}$       B)  $\frac{7}{8008}$       C)  $\frac{3}{7}$       D)  $\frac{3}{286}$       E)  $\frac{283}{286}$

Answer # 14:

$$1 - P(\text{No Women}) = 1 - \frac{C(9, 6)}{C(16, 6)} = \frac{283}{286}$$

# EXAM A — PART 2

Name SOLUTIONS ID Number \_\_\_\_\_ Instructor \_\_\_\_\_

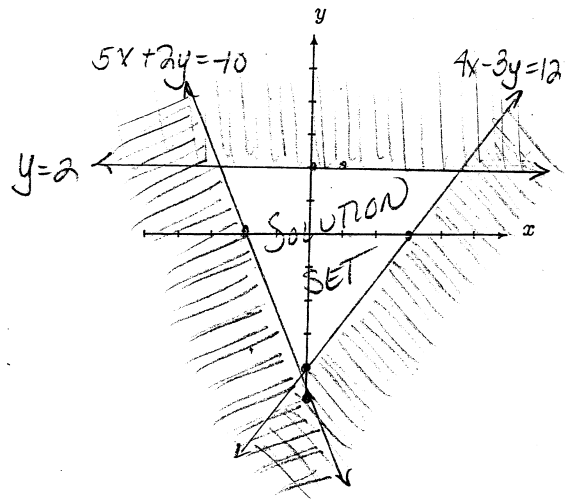
Part II: No calculators allowed. You may return to Part 1 without using a calculator. Do not use decimals.

1. Solve the system of inequalities. Show all work for the shading and clearly label the solution set. (9 points)

$$4x - 3y \leq 12, \quad 5x + 2y \geq -10, \quad y \leq 2$$

$4x - 3y = 12$	$5x + 2y = -10$	$y = 2$
$(0, -4)$	$(0, -5)$	$(0, 2)$
$(3, 0)$	$(-2, 0)$	$(1, 2)$

test $(0, 0)$	test $(0, 0)$	test $(0, 0)$
$0 \leq 12$	$0 \geq -10$	$0 \leq 2$
TRUE	TRUE	TRUE



2. SET UP a linear program for the following word problem. DO NOT SOLVE.

A manufacturer of office and home furniture makes desks and chairs. Each desk requires 1 hour for assembly, 0.75 hours for finishing, and 0.3 hours for inspection and packing. Each chair requires 0.5 hours for assembly, 0.4 hours for finishing, and 0.2 hours for inspection and packaging. Each week the manufacturer has available up to 250 hours of time for assembling desks and chairs, up to 220 hours for finishing desks and chairs, and up to 110 hours each week for inspecting and packaging the desks and chairs. Based on the demand for its desks and chairs, the manufacturer makes at least three times as many chairs as desks each week. The manufacturer's profit is \$60 for each chair and \$95 for each desk. How many chairs and desks should the manufacturer make each week in order to make the largest possible profit? (12 points)

$x = \# \text{ OF DESKS}$	$\text{MAX } P = 95x + 60y$	
$y = \# \text{ OF CHAIRS}$	$x + .5y \leq 250$	ASSEMBLY
(MADE EACH WEEK)	$.75x + .4y \leq 220$	FINISHING
	$.3x + .2y \leq 110$	INSPECTING + PACKAGING
	$y \geq 3x$	
	$x \geq 0$	

### Final Exam A – Part 2

3. The linear programming problem: Maximize and Minimize  $P = 24x - 21y$

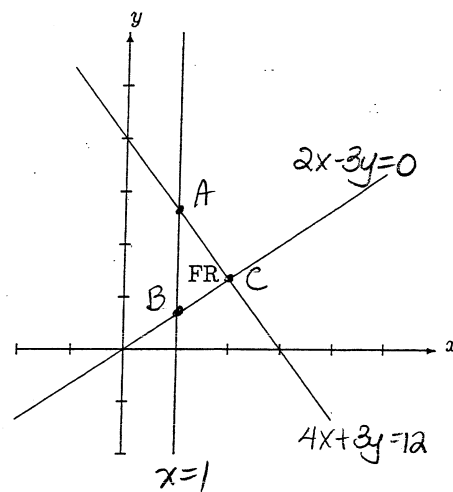
subject to:  $4x + 3y \leq 12$ ,  $2x - 3y \leq 0$ ,  $x \geq 1$

has the feasible region (FR) shown at the right.

(a) Determine the coordinates of all corner points. Show your work. (6 points)

A)  $x=1$   $(1, \frac{8}{3})$   
 $4+3y=12$   
 $3y=8$   
 $y=\frac{8}{3}$

B)  $x=1$   $(1, \frac{2}{3})$   
 $2-3y=0$   
 $-3y=-2$   
 $y=\frac{2}{3}$



C)  $4x+3y=12$   $4x+3y=12$   $4x+3y=12$   $(2, \frac{4}{3})$   
 $2x-3y=0$   $-2(2x-3y=0) \rightarrow -4x+6y=0$   
 $6x=12$   $9y=12$   
 $x=2$   $y=\frac{12}{9}=\frac{4}{3}$

(b) Determine the solution to the problem. (3 points)

CP	$24x - 21y =$
$(1, \frac{8}{3})$	$24(1) - 21(\frac{8}{3}) = 24 - 56 = -32$
$(1, \frac{2}{3})$	$24(1) - 21(\frac{2}{3}) = 24 - 14 = 10$
$(2, \frac{4}{3})$	$24(2) - 21(\frac{4}{3}) = 48 - 28 = 20$

**SOLUTION:** the maximum value is 20 and it occurs at the corner point  $(2, \frac{4}{3})$ ;

the minimum value is -32 and it occurs at the corner point  $(1, \frac{8}{3})$ .

Final Exam A – Part 2 – Multiple Choice

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ANSWERS BELOW ↓

15. Find the product:  $\begin{pmatrix} -2 & 5 & 10 \\ -8 & -1 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$

- A)  $\begin{pmatrix} -10 & -40 \\ -15 & 3 \\ 40 & 16 \end{pmatrix}$     B)  $( 15 \quad -21 )$     C)  $\begin{pmatrix} 15 \\ -21 \end{pmatrix}$     D)  $\begin{pmatrix} 65 \\ -21 \end{pmatrix}$

Answer # 15:  C

16. What is the inverse matrix for  $\begin{pmatrix} -8 & -3 \\ 7 & 4 \end{pmatrix}$

- A)  $\begin{pmatrix} -\frac{8}{53} & -\frac{7}{53} \\ \frac{3}{53} & \frac{4}{53} \end{pmatrix}$     B)  $\begin{pmatrix} -\frac{4}{53} & -\frac{3}{53} \\ \frac{7}{53} & \frac{8}{53} \end{pmatrix}$     C)  $\begin{pmatrix} -\frac{4}{11} & \frac{3}{11} \\ -\frac{7}{11} & \frac{8}{11} \end{pmatrix}$     D)  $\begin{pmatrix} -\frac{4}{11} & -\frac{3}{11} \\ \frac{7}{11} & \frac{8}{11} \end{pmatrix}$

Answer # 16:  D

17.  $C(7,5) =$

- A) 2520    B) 21    C) 210    D) 504

Answer # 17:  B

18.  $P(6,4) =$

- A) 120    B) 15    C) 360    D) 5

Answer # 18:  C