Final Exam A - Part 1

1. The message

$$-9, 10, 8, -13, -39, 55, -18, 26, -8, 15, -15, 20, -43, 58$$

was encoded using the matrix $M = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$ and the following coding scheme:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
0	1	−1	2	−2	3	−3	4	−4	5	−5	6	−6	7	−7	8	−8	9

S	T	U	V	W	X	Y	Z	blank	$	,	.	!	?
−9	10	−10	11	−11	12	−12	13	−13	14	−14	15	−15	16

Showing the matrix multiplication that must be performed, decode this message. (6 points)

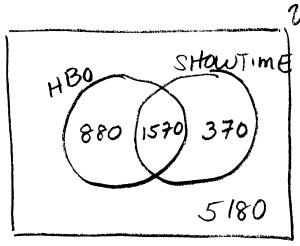
$\det = 2(-4) - (-3)(3) = 1 \qquad M^{-1} = \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix}$

$$\begin{pmatrix} -9 & 10 \\ 8 & -13 \\ -39 & 55 \\ -18 & 26 \\ -8 & 15 \\ -15 & 20 \\ -43 & 58 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -7 \\ 7 & -2 \\ -9 & -7 \\ -6 & -2 \\ -13 & 6 \\ 0 & -5 \\ -2 & -13 \end{pmatrix}$$

The message is: LONESOME LAKE

A beautiful place in the White Mountains

2. A cable TV company has 8,000 subscribers in a suburban community. The company offers two premium channels, HBO and Showtime. If 2,450 subscribers receive HBO, 1,940 receive Showtime, and 5,180 do not receive any premium channel, how many subscribers receive both HBO and Showtime? (4 points)



$U = 8000$

$n(H \cup S) = n(H) + n(S) - n(H \cap S)$

$8820 = 2450 + 1940 - n(H \cap S)$

$n(H \cap S) = 1570$

Final Exam A - Part 1

3. A furniture company manufactures pine tables and chairs at factories in Wytheville and Andersen. Each table requires 2 hours of cutting and milling, 1 hour of assembly, and 2 hours of finishing. Each chair requires 1.5 hours of cutting and milling, 1 hour of assembly, and 0.5 hours of finishing.

   (a) Represent the table and chair labor requirements as a $3x2$ time matrix T. Label each row and column. (2 points)

$$T = \begin{matrix} \\ \text{cutting} \\ \text{assembly} \\ \text{finishing} \end{matrix} \begin{matrix} \begin{matrix} \text{table} & \text{chair} \end{matrix} \\ \begin{pmatrix} 2 & 1.5 \\ 1 & 1 \\ 2 & .5 \end{pmatrix} \end{matrix}_{3\times 2}$$

   (b) At the Wytheville factory, the per-hour labor costs are \$9 for cutting and milling work, \$14 for assembly work, and \$13 for finishing work. At the Andersen factory, the per-hour labor costs are \$10 for cutting and milling work, \$13 for assembly work, and \$12 for finishing work. Represent the factory per-hour labor costs as a cost matrix C such that when you multiply these two matrices (time and costs), *the resulting product matrix will give the production costs of the tables and chairs at each factory.* Label each row and column. (4 points)

$$C = \begin{matrix} \\ \text{Wytheville} \\ \text{Andersen} \end{matrix} \begin{matrix} \begin{matrix} \text{Cutting} & \text{Assembly} & \text{Finishing} \end{matrix} \\ \begin{pmatrix} 9 & 14 & 13 \\ 10 & 13 & 12 \end{pmatrix} \end{matrix}_{2\times 3}$$

   (c) **Write the matrix equation and calculate the product.** What does it cost to produce a chair at the Andersen factory? (5 points)

$$CT = \begin{matrix} \\ \text{Wytheville} \\ \text{Andersen} \end{matrix} \begin{matrix} \begin{matrix} \text{table} & \text{chair} \end{matrix} \\ \begin{pmatrix} 58 & 34 \\ 57 & 34 \end{pmatrix} \end{matrix}$$

It cost \$34 to produce a chair at the Andersen factory

Final Exam A - Part 1

4. Consider the system of equations $\begin{aligned} 6x - 3y &= -7 \\ 5x + 4y &= 3 \end{aligned}$.

(a) Express this system as a matrix equation. (2 points)

$$\begin{pmatrix} 6 & -3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 3 \end{pmatrix}$$

A B

(b) **Showing the matrices to be multiplied**, solve your matrix equation for x and y. (6 points)

$\det = 6(4) - (-3)(5) = 39$

$$\begin{pmatrix} 4/39 & 3/39 \\ -5/39 & 6/39 \end{pmatrix} \begin{pmatrix} -7 \\ 3 \end{pmatrix} = \begin{pmatrix} -19/39 \\ 53/39 \end{pmatrix} \quad \text{or} \quad A^{-1}B = \begin{pmatrix} -19/39 \\ 53/39 \end{pmatrix}$$

A^{-1} B

$\left(-\frac{19}{39}, \frac{53}{39}\right)$

5. A survey of retirees in Arizona who golf or fish every day found that 60% of those who played golf one day switched to fishing the next and that 90% of those who went fishing switched to golf the next day.

(a) Construct the transition (stochastic) matrix for this Markov chain. (4 points)

$$\begin{matrix} & \text{Golf} & \text{Fishing} \\ \text{Golf} \\ \text{Fishing} \end{matrix} \begin{pmatrix} .4 & .9 \\ .6 & .1 \end{pmatrix} = T$$

(b) Suppose that 30% of these retirees are golfing today. Give the initial distribution matrix. (1 point)

$$\begin{matrix} \text{Golf} \\ \text{Fishing} \end{matrix} \begin{pmatrix} .3 \\ .7 \end{pmatrix} = d_0$$

(c) **Showing the matrices to be multiplied**, answer the following question. What percentage of these retirees will be fishing the day after tomorrow? (4 points)

$$Td_0 = \begin{matrix} \text{Golf} \\ \text{Fishing} \end{matrix} \begin{pmatrix} .75 \\ .25 \end{pmatrix} = d_1 \quad \text{or} \quad T^2 d_0 = \begin{matrix} \text{Golf} \\ \text{Fishing} \end{matrix} \begin{pmatrix} .525 \\ .475 \end{pmatrix} = d_2$$

$$Td_1 = \begin{matrix} \text{Golf} \\ \text{fishing} \end{matrix} \begin{pmatrix} .525 \\ .475 \end{pmatrix} = d_2$$

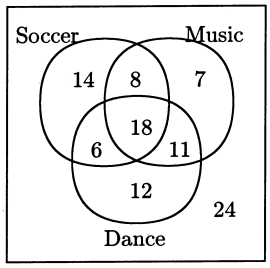
47.5% will be fishing the day after tomorrow.

Final Exam A – Part 1

Multiple Choice Section

Put the *letter* of the correct answer in the box at the right side of the page AND fill in the letter on the bubble sheet.
This is the only way to get credit for your answer. **ANSWERS BELOW ⇓**

A survey of the parents of 100 third graders asked what after school activities their child participated in. The information collected is given in the Venn diagram below. Use this diagram to answer questions #2-4.



1. **FILL IN THE LETTER 'A' FOR #1 ON THE BUBBLE SHEET.**

Answer # 1: A

2. What is the probability a child in the survey participates in soccer or dance?

A) $\frac{32}{100}$ B) $\frac{45}{100}$ C) $\frac{69}{100}$ D) $\frac{24}{100}$ E) $\frac{6}{100}$

Answer # 2: C

3. What is the probability a child in the survey takes music lessons?

A) $\frac{7}{100}$ B) $\frac{44}{100}$ C) $\frac{18}{100}$ D) $\frac{26}{100}$ E) $\frac{15}{100}$

Answer # 3: B

4. What is the probability that a child in the survey participates in music and dance but not soccer?

A) $\frac{30}{100}$ B) $\frac{29}{100}$ C) $\frac{11}{100}$ D) $\frac{40}{100}$ E) $\frac{44}{100}$

Answer # 4: C

5. How many distinct arrangements are there of the letters in the word EXCELLENCE?

A) 37,800 B) 604,800 C) 3,628,800 D) 151,200 E) 75,600

Answer # 5: A

$10!/(4!\,2!\,2!)$

Final Exam A – Part 1 – Multiple Choice

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6. How many 5-letter code words are possible from the first 7 letters of the alphabet if adjacent letters must be different?

A) 2520 B) 1512 C) 6300 D) 9072 E) 16,807 Answer # 6: D

7 6 6 6 6

7. A catering service offers 8 appetizers, 10 main courses, and 7 desserts. A banquet committee is to select 3 appetizers, 4 main courses, and 2 desserts. How many ways can this be done?

A) 24 B) 560 C) 19,200 D) 246,960 E) 287 Answer # 7: D

$C(8,3) \cdot C(10,4) \cdot C(7,2)$

$\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6}{2 \cdot 1}$

8. Ten students apply for the position of grader in mathematics. One grader is to be assigned to each of four teachers. In how many different ways can the teachers be assigned a grader if no student grades for more than one teacher?

A) 210 B) 10,000 C) 5040 D) 25,200 E) 151,200 Answer # 8: C

10 9 8 7

9. In how many different ways can four men and three women be seated in a row if no one sits next to a member of the same gender?

A) 144 B) 5040 C) 2880 D) 576 E) 186,624 Answer # 9: A

4 3 3 2 2 1 1 = 144
M W M W M W M

10. A radio announcer says the odds that the Yankees will beat the Orioles are 7:4. What does the announcer believe is the probability that the Yankees will beat the Orioles?

A) $\frac{4}{7}$ B) $\frac{4}{11}$ C) $\frac{7}{4}$ D) $\frac{11}{7}$ E) $\frac{7}{11}$ Answer # 10: E

Final Exam A – Part 1 – Multiple Choice

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A jar contains 8 red and 7 purple marbles. A sample of 5 are randomly selected from the jar. Use this information to answer questions #11 – #14.

11. How many different samples are possible?

A) $P(15,5)$ B) $C(15,5)$ C) $C(8,5)$ D) 8!7! E) $\frac{15!}{8!7!}$ Answer # 11: B

12. How many of these samples contain 3 red and 2 purple marbles?

A)77 B) 14,112 C) 1176 D) 63 E) 9075 Answer # 12: C

13. What is the probability that a sample contains 3 red and 2 purple marbles?

A) $\frac{1}{39}$ B) $\frac{56}{143}$ C) $\frac{7}{2145}$ D) $\frac{3}{143}$ E) $\frac{77}{360360}$ Answer # 13: B

14. What is the probability the sample contains at least 1 red marble?

A) $\frac{8}{3003}$ B) $\frac{87}{143}$ C) $\frac{21}{360360}$ D) $\frac{142}{143}$ E) $\frac{21}{3003}$ Answer # 14: D

Department of Mathematics **Northeastern University**

MTHU115 Final Exam Part 2 — Spring 2004

EXAM A — PART 2

Name_____ **ID Number**_____ **Instructor**_____

Part II: No calculators allowed. You may return to Part 1 without using a calculator. Do not use decimals.

1. **SET UP** a linear program for the following word problem. **DO NOT SOLVE.**
Fastechnical, a marine fabricating company, produces aluminum t-tops and radar arches. Each t-top requires 32 feet of $1\frac{1}{4}''$ pipe, 40 feet of $1\frac{1}{2}''$ pipe and 30 feet of $1''$ pipe. Each radar arch requires 30 feet of $1\frac{1}{4}''$ pipe, 30 feet of $1\frac{1}{2}''$ pipe and 18 feet of $1''$ pipe. The profit made on a radar arch is \$800 and the profit on a t-top is \$1050. They have in stock 2000 feet of $1\frac{1}{4}''$ pipe, 2500 feet of $1\frac{1}{2}''$ pipe and 1800 feet of $1''$ pipe. They also have an order to fill for 2 radar arches. How many of each item should the company produce to maximize their earnings?
(12 points)

x = # of t-tops
y = # of radar arches

$MAX\ E = 1050x + 800y$ Earnings

$32x + 30y \leq 2000$ $1\frac{1}{4}''$ Pipe
$40x + 30y \leq 2500$ $1\frac{1}{2}''$ Pipe
$30x + 18y \leq 1800$ $1''$ Pipe
$x \geq 0$
$y \geq 2$

2. **Showing your work,** shade the region in the Venn diagram below that represents $(R \cup T') \cap (S \cap R')'$

$R = \{1,2,4,5\}$ $T' = \{1,2,3,8\}$

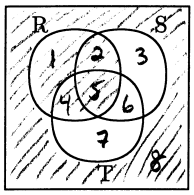
$R \cup T' = \{1,2,3,4,5,8\}$

$S = \{2,3,5,6\}$ $R' = \{3,6,7,8\}$

$S \cap R' = \{3,6\}$

$(S \cap R')' = \{1,2,4,5,7,8\}$

$(R \cup T') \cap (S \cap R')' = \{1,2,4,5,8\}$



(6 points)

Final Exam A – Part 2

3. The linear programming problem: Maximize and Minimize $P = 14x + 21y$

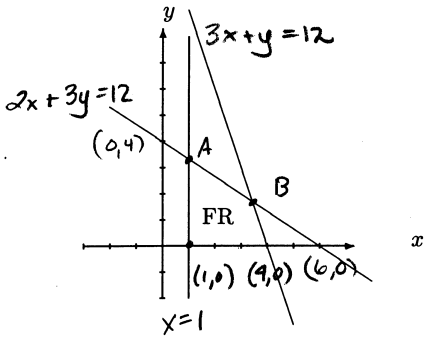
(0,4) (6,0)

subject to: $2x + 3y \leq 12, \quad 3x + y \leq 12, \quad x \geq 1, \quad y \geq 0$

has the feasible region (FR) shown at the right.

(a) Determine the coordinates of all corner points. Show your work. (6 points)

A, B, (1,0), (4,0)



A) $x = 1 \quad (1, 10/3)$

$2x + 3y = 12$

$3y = 10$

$y = 10/3$

B) $-3(3x + y = 12) \rightarrow -9x - 3y = -36$

$2x + 3y = 12$

$+$

$-7x = -24$

$x = \frac{24}{7}$ $\quad (24/7, 12/7)$

$2(3x + y = 12) \rightarrow 6x + 2y = 24$

$-3(2x + 3y = 12) \rightarrow -6x - 9y = -36$

$+$

$-7y = -12$

$y = 12/7$

(b) Determine the solution to the problem. (4 points)

CP's	$14x + 21y$
(1,0)	$14(1) + 21(0) = 14$
(4,0)	$14(4) + 21(0) = 56$
$(1, 10/3)$	$14(1) + 21(10/3) = 14 + 70 = 84$
$(24/7, 12/7)$	$14(24/7) + 21(12/7) = 48 + 36 = 84$

SOLUTION: the maximum value is 84 and it occurs at the corner point (1 , $10/3$); $(24/7, 12/7)$

the minimum value is 14 and it occurs at the corner point (1 , 0).

Final Exam A – Part 2 – Multiple Choice

Put the *letter* of the correct answer in the box at the right side of the page AND fill in the letter on the bubble sheet.
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15. Find the product: $\begin{pmatrix} 10 & -4 \\ 5 & 6 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

A) $\begin{pmatrix} -62 & -7 & 36 \end{pmatrix}$ B) $\begin{pmatrix} -62 \\ -7 \\ 36 \end{pmatrix}$ C) $\begin{pmatrix} -50 & -12 \\ -25 & 18 \\ 15 & 21 \end{pmatrix}$ D) $\begin{pmatrix} -38 \\ -43 \\ -6 \end{pmatrix}$ Answer # 15: B

16. What is the inverse matrix for $\begin{pmatrix} 7 & -3 \\ 5 & -6 \end{pmatrix}$

A) $\begin{pmatrix} -\frac{7}{27} & \frac{5}{27} \\ -\frac{3}{27} & \frac{6}{27} \end{pmatrix}$ B) $\begin{pmatrix} -\frac{6}{27} & \frac{3}{27} \\ -\frac{5}{27} & -\frac{7}{27} \end{pmatrix}$ C) $\begin{pmatrix} \frac{6}{27} & -\frac{3}{27} \\ \frac{5}{27} & -\frac{7}{27} \end{pmatrix}$ D) $\begin{pmatrix} \frac{6}{57} & -\frac{3}{57} \\ \frac{5}{57} & -\frac{7}{57} \end{pmatrix}$ Answer # 16: C

17. $P(7,3) =$

A) 210 B) 2520 C) 35 D) 336 Answer # 17: A

$7 \cdot 6 \cdot 5 = 210$

18. $C(9,4) =$

A) 3024 B) 2520 C) 756 D) 126 Answer # 18: D

$\frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$