

MTHU115 Final Exam Review - Part One - Calculator Section

1. The message

19, 13, 54, 37, -35, -23, -14, -8, 48, 33, -73, -52, 49, 37, -2, -1, 139, 98

was encoded using the matrix $M = \begin{pmatrix} -3 & -2 \\ 7 & 5 \end{pmatrix}$ and the following coding scheme:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	
1	-1	2	-2	3	-3	4	-4	5	-5	6	-6	7	-7	8	-8	9	-9	
S	T	U	V	W	X	Y	Z	blank	\$,	.	!	?					
10	-10	11	-11	12	-12	13	-13	14	-14	15	-15	16	-16					

(a) What matrix is needed to decode this message?

(b) What is the message?

The message is: _____

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2. Consider the system of equations
$$\begin{array}{rcl} 9x - 8y & = & -5 \\ -8x + 5y & = & 6 \end{array} .$$

(a) Express this system as a matrix equation.

(b) Showing the matrices to be multiplied, solve your matrix equation for x and y .

3. The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry, Mucho Mocha, and Almond Delight. The company produces its products in San Diego, Mexico City, and Managua using two main ingredients: chocolate and sugar.

(a) Each kilogram of Cheery Cherry requires .5 kg of sugar and .2 kg of chocolate; each kilogram of Mucho Mocha requires .4 kg sugar and .3 kg chocolate; and each kilogram of Almond Delight requires .3 kg of sugar and .3 kg of chocolate. Put this information into a 2×3 matrix, labelling the rows and columns.

(b) The cost of 1 kg of sugar is \$4 in San Diego, \$2 in Mexico City, and \$1 in Managua. The cost of 1 kg of chocolate is \$3 in San Diego, \$5 in Mexico City and \$7 in Managua. Put this information into a matrix in such a way that when you multiply it with your matrix from part a, you get a matrix representing the ingredient cost of producing each type of candy in each city.

(c) Multiply the matrices from parts a and b, labeling the product matrix.

(d) From part c, what is the combined sugar-and-chocolate cost to produce 1 kg of Mucho Mocha in Managua?

4. How many distinct arrangements are there of the letters in the word INTELLIGENCE?

5. The first aid kit in the emergency room of a local hospital is stocked with one of 3 brands of cough medicine, one of 4 brands of antiseptic cream, one of 5 brands of painkiller, and one of 8 styles of bandages. How many different first aid kits containing one of each of these brand of item are possible?

6. In how many ways can a supermarket chain select 3 out of 15 possible sites for the construction of new supermarkets?

7. In how many ways can four married couples attending a concert be seated in a row of eight seats if:
 - (a) There are no restrictions?

 - (b) Each married couple is seated together?

8. The C&J Realty Company has received 12 inquiries from prospective home buyers. In how many ways can the inquiries be directed to four of the firm's real estate agents if each agent handles three inquiries?

9. At a national convention, the Credentials Committee consists of five people. If there are nine men and six women to choose from, in how many ways
 - (a) can a committee of 5 people be chosen?

 - (b) can a committee of 3 men and 2 women be chosen?

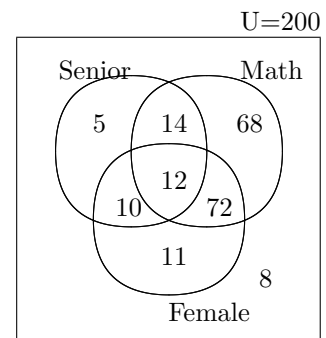
 - (c) can a committee of at least four women be chosen?

 - (d) can a committee of at least one man be chosen?

10. If $n(A \cup B) = 36$, $n(A) = 23$, and $n(A \cap B) = 17$, what is $n(B)$?

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11. A class of 30 students was given a diagnostic test on the first day of a mathematics course. At the end of the semester, only 2 of the 21 students who had passed the diagnostic test failed the course. A total of 23 students passed the course. Draw an appropriate Venn diagram for this problem and answer the following question. How many students managed to pass the course even though they had failed the diagnostic test?
12. 30 businesses were asked which overnight express mail services they used. When the responses were tallied, they showed that: 16 used Federal Express (FedEx), 14 used United Parcel Service (UPS), 11 used the U.S. Postal Service (USPS), 2 used all three services, 6 used only FedEx, 5 used only UPS, 4 used UPS and USPS but not FedEx. Construct a complete Venn Diagram for this population. Check your arithmetic, there will be no partial credit given.
13. Out of 200 high school students it was found that:
 41 students are seniors, 166 students are taking a mathematics course, 105 students are female, 26 students are seniors taking a mathematics course, 84 students are females taking a mathematics course, 22 students are female seniors, and 12 students are female seniors taking a mathematics course.
 Use the Venn Diagram below to answer the questions.



- (a) What is the probability the student is a non-senior taking a math course?
- (b) What is the probability the student is a male senior that is not taking a math course?
- (c) What is the probability the student is a female or a senior?

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14. A standard deck of 52 cards consists of 4 suits (Hearts, Diamonds, Spades, and Clubs). Hearts and Diamonds are red cards. Each suit has thirteen cards (Ace through King). So there are four of each card in the deck. Given a well-shuffled standard deck of cards, find the probability that a card drawn at random will be a red or a face card (Jacks, Queens, and Kings).

15. Given a well-shuffled standard deck of cards, find the probability that three cards drawn at random will be all aces.

16. Suppose two fair dice are rolled. What is the probability that the sum of the numbers rolled is greater than 3?

17. If the odds in favor of a particular horse's winning a race are 5 to 7, what is the probability that the horse will win the race?

18. Within a large metropolitan area, 20% of the commuters currently use the public transportation system, whereas the remaining 80% commute by automobile. The city has recently revitalized and expanded its public transportation system. It is expected that 6 months from now 20% of those now using public transportation will commute by car and 80% will continue to use public transportation. At the same time, it is expected that 30% of those who are now commuting to work by car will switch to public transportation and 70% will continue to commute by car.
 - (a) Construct the stochastic (transition) matrix for the Markov chain that describes the change in the mode of transportation used by these commuters.

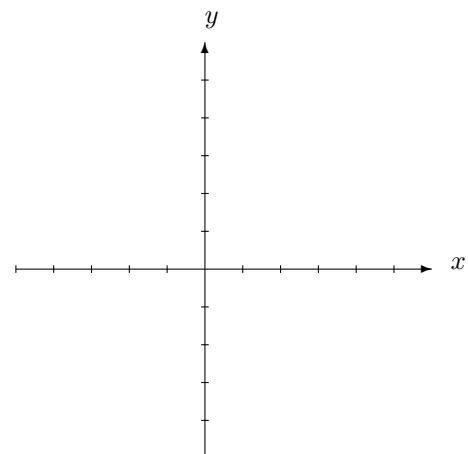
 - (b) Find the 2×1 initial distribution matrix for this Markov chain.

 - (c) What percentage of the commuters are expected to use public transportation 6 months from now?

MTH1101 Final Exam Review - Part Two - Non-Calculator Section

19. Solve the system of inequalities. **Show all work for the shading and clearly label the solution set.**

$$x - y \leq 4, \quad 3x + 2y > 0, \quad x \geq 0$$



20. Find $3Y - 2X$ where $X = \begin{pmatrix} -6 & 8 & -3 \\ 4 & 2 & -7 \end{pmatrix}$ and $Y = \begin{pmatrix} 5 & -4 & 3 \\ -5 & 8 & -2 \end{pmatrix}$.

21. Find the product: $\begin{pmatrix} -5 & 3 & 8 \\ 5 & -2 & 3 \end{pmatrix} \begin{pmatrix} 9 & 4 \\ -3 & 6 \\ 3 & -6 \end{pmatrix}$

22. What is the inverse matrix for $\begin{pmatrix} -7 & -5 \\ 4 & 6 \end{pmatrix}$

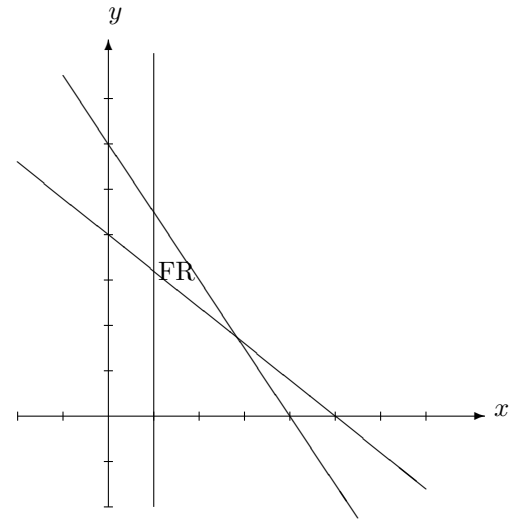
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23. The linear programming problem: Maximize and Minimize $P = 7x + 35y$

subject to: $x \geq 1$, $4x + 5y \geq 20$, $3x + 2y \leq 12$

has the feasible region (FR) shown at the right.

(a) Determine the coordinates of all corner points.

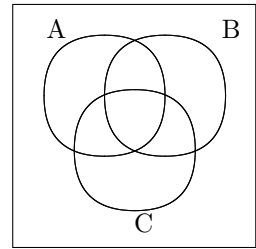


(b) Determine the solution to the problem.

SOLUTION: the maximum value is _____ and it occurs at the corner point (,);
the minimum value is _____ and it occurs at the corner point (,).

Part II: No calculators allowed. You may return to Part I without using a calculator.

24. Shade the region in the Venn diagram below that represents $(A' \cap C') \cup (B \cap C)$



25. **SET UP** a linear program for the following word problem. **DO NOT SOLVE.**

A small country can grow only two crops for export, coffee and cocoa. The country has 500,000 hectares of land available for the crops. Long-term contracts require that at least 100,000 hectares be devoted to coffee and at least 200,000 hectares to cocoa. Cocoa must be processed locally, and production bottlenecks limit cocoa to 270,000 hectares. Coffee requires two workers per hectare, with cocoa requiring five. No more than 1,750,000 people are available for working with these crops. Coffee produces a profit of \$220 per hectare and cocoa a profit of \$550 per hectare. How many hectares should the country devote to each crop in order to maximize profit?

26. Let $U = \{a, b, c, d, e, f, g\}$, $A = \{c, d, e, f\}$, $B = \{a, e, f\}$, $C = \{a, d, g\}$. Find each of the following:

(a) $(B' \cap C) \cup A$

(b) $(A \cup B)' \cap C$

(c) $(B' \cap C)' \cup A$

27. Let $U = \{\text{all students at NU}\}$, $A = \{\text{all athletes}\}$, $M = \{\text{all males}\}$, and $C = \{\text{all students who commute}\}$. Express each of the following sets in terms of unions, intersections and/or complements:

(a) $\{\text{all athletes or resident students at NU}\}$

(b) $\{\text{all female athletes at NU}\}$

28. Evaluate each of the following. You must show how you arrived at your answers.

(a) $P(7, 3)$

(b) $C(8, 4)$