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Name: _____

MTH U115

Applications of Algebra

Spring 2005

MIDTERM EXAM

Instructions: Put your name in the blanks above. Put your final answers to each question in the designated spaces on these pages. There are **no calculators** allowed during this exam. For full credit, **show all of your work**. If there is not enough room, use another sheet of paper.

- (1) **SET UP** a linear program to solve the following problem. Be sure to identify the variables, **ALL** the constraints, and the objective function. **DO NOT SOLVE**.

A small generator burns two types of fuel—low sulfur and high sulfur—to produce electricity. For each hour of use, each gallon of low sulfur emits 4 units of sulfur dioxide, generates 5 kilowatts of electricity, and costs \$2.50, while each gallon of high sulfur emits 8 units of sulfur dioxide, generates 12 kilowatts of electricity, and costs \$3.25. The supply of low-sulfur fuel is limited to 6,000 gallons, and the supply of high sulphur fuel to 2,000 gallons. The EPA insists that the maximum amount of sulfur dioxide that may be emitted per hour is 70 units. Suppose that at least 90 kilowatts must be generated per hour to meet demand. How many gallons of low sulfur and how many gallons of high sulfur should be used hourly in order to minimize the cost of the fuel used?

(2) Use the addition method to find the point of intersection of the lines

$$4x + 6y = 3$$

$$5x - 3y = 2$$

DO NOT express your answer in digital form. **DO NOT** graph the lines.

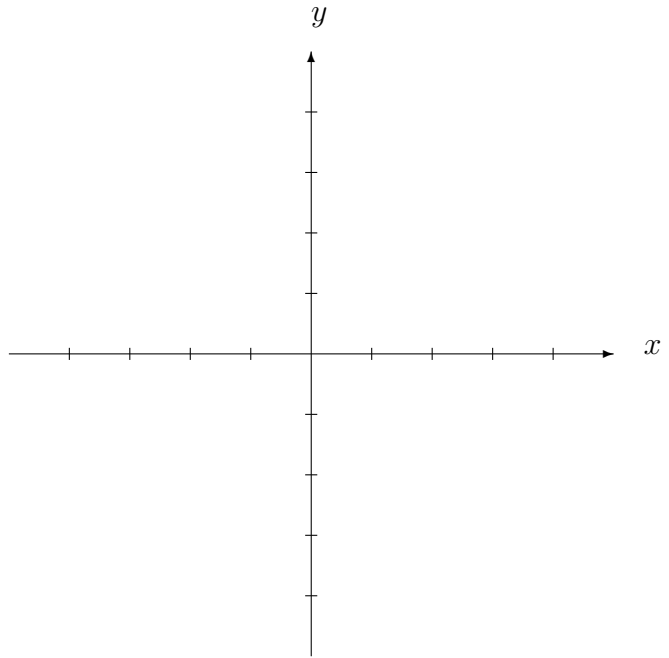
(3) Given the system of equations
$$\begin{cases} 7x + 4y = -1 \\ 8x + 5y = 2 \end{cases}$$

(a) Express the system in matrix form.

(b) Solve the system by using the inverse of the coefficient matrix.

(4) Solve the following system of linear inequalities AND shade the region.

$$y \leq 4, \quad 2x - y > 0, \quad x + 3y \geq 3$$



(5) Find $3A - 4B$, where $A = \begin{bmatrix} 5 & 2 & 0 & -3 \\ 1 & -4 & 2 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 2 & 4 \\ -5 & 1 & 0 & -2 \end{bmatrix}$.

(6) How many different license plates consisting of one letter, followed by a three-digit number are there?

- (7) A company has two factories, each with three production lines, I, II, and, III. In factory 1, line I produces 30 items per hour, line II produces 50 items per hour, and line III produces 40 items per hour. In factory 2, line I produces 60 items per hour, line II produces 20 items per hour, and line III produces 25 items per hour.

At each factory during the first week, line I runs for 40 hours, II runs for 20 hours, and III runs for 30 hours. At each factory during the second week, line I runs for 15 hours, II runs for 30 hours, and III runs for 40 hours.

- (a) Write a matrix A which has one row for each factory and whose entries give the items per hour produced in each production line.

- (b) Write a matrix B which has a column for each week the production line runs and whose entries give the number of hours each production line runs.

- (c) Find the product AB .

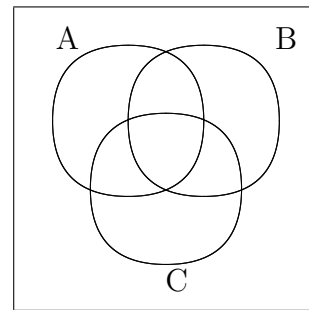
- (d) How many items were produced in the second week by factory 1?

- (8) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 3, 4, 5\}$, $B = \{0, 3, 6, 9\}$, $C = \{1, 6, 8\}$.
Find each of the following:

(a) $A' \cap C'$

(b) $(C \cup B)' \cap A'$

- (9) Shade the region in the Venn diagram below that represents $(A \cup C') \cap B'$.



- (10) Find $n(S)$, given that $n(T) = 22$, $n(S \cap T) = 9$, and $n(S \cup T) = 40$.

- (11) Let $U = \{\text{all employees in a hospital}\}$, $N = \{\text{all nurses}\}$, $D = \{\text{all doctors}\}$, $A = \{\text{all administrators}\}$, $F = \{\text{all female employees}\}$.

- (a) Express the following set in terms of unions, intersections and/or complements:

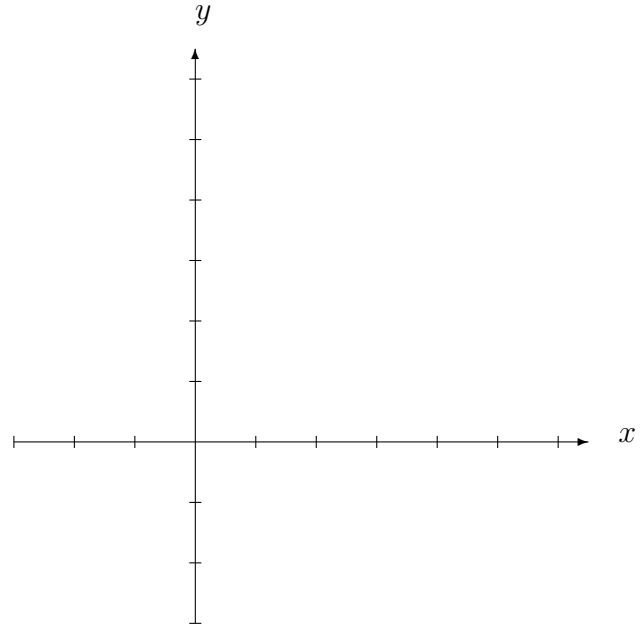
{ all male doctors who are not administrators }

- (b) Describe in a sentence the set defined by $(F \cap D) \cup (F' \cap N)$.

(12) **SOLVE** the following linear programming problem:

Maximize and minimize $F = 10x + 3y$

subject to the constraints: $2x + 3y \geq 7$, $x \leq 5$, $y \geq 0$, $y \leq 4$



Solution:

- The maximum value of F is _____, and it occurs at the point (____ , ____).
- The minimum value of F is _____, and it occurs at the point (____ , ____).