19. **a.** No; there are blue-eyed males.

b. Yes; a brown-eyed female doesn't have blue eyes.

c. Yes; a brown-eyed female isn't male.

20. **a.** $E \cup F$ = "blue eyes or male"

b. $E \cap G = \varnothing$

c. E' = "not blue eyes"

d. F' = "not male" = "female"

e. $(G \cup F) \cap E = (G \cap E) \cup (F \cap E) = \varnothing \cup (F \cap E) = F \cap E$ = "blue eyes and male"

f. $G' \cap E = E$ = "blue eyes"

21. Length of line in number of persons: the set of nonnegative integers.

22. Time in minutes: the set of positive numbers.

23. Time in minutes: the set of nonnegative numbers.

24. Pairs of data from Exercises 21 and 22:
{(a, b), where a is a nonnegative integer and b is a positive number}

25. The set of all possible triplets of three-digit numbers:
{(000, 000, 000), (000, 000, 001), … (999, 999, 999)}
E' = the event that at least one number is odd
$E \cap F$ = the event that all numbers are even and are more than 699

26. **a.** For the man, listing older child first:
{(boy, boy), (boy, girl)}
For the woman, listing older child first:
{(boy, boy), (boy, girl), (girl, boy)}

b. E = {(boy, boy)}

c. F = {(boy, boy)}

27. **a.** 6 suspects × 6 weapons × 9 rooms = 324 outcomes

b. $E \cap F$ = "The murder occurred in the library with a gun."

c. $E \cup F$ = "Either the murder occurred in the library, or it was perpetrated with a gun."

Exercises 6.3

1. **a.** $\dfrac{46,277}{774,746}$

b. $\dfrac{46,277+1855}{774,746} = \dfrac{48,132}{774,746}$

c. $\dfrac{774,746-48,132}{774,746} = \dfrac{726,614}{774,746}$

2. a. $\frac{1}{4}$

b. 3 out of the 4 outcomes are in *S*, so the probability is $\frac{3}{4}$.

3. a. E = "the numbers add up to 8" = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}
$\Pr(E) = \frac{5}{36}$

b. $\Pr(\text{sum is } 2) = \Pr((1, 1)) = \frac{1}{36}$; $\Pr(\text{sum is } 3) = \Pr((1, 2)) + \Pr((2, 1)) = \frac{2}{36}$
$\Pr(\text{sum is } 4) = \Pr((1, 3)) + \Pr((2, 2)) + \Pr((3, 1)) = \frac{3}{36}$
The probability that the sum is less than 5 is $\frac{1}{36}+\frac{2}{36}+\frac{3}{36}=\frac{1}{6}$.

4. $\frac{6}{50}=\frac{3}{25}$

5. $\frac{1}{38}+\frac{1}{38}=\frac{2}{38}=\frac{1}{19}$

6. a. $\frac{3}{9}=\frac{1}{3}$

b. $\frac{5}{9}$

c. The probability of a 1, 2, 3, 5, 7, or 9 is $\frac{6}{9}=\frac{2}{3}$.

7. a. $1-\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{6}$

b. The probability is $\frac{1}{6}$. Thus $a = 1$ and $a + b = 6$, so $b = 5$. The odds are 1 to 5.

8. a. No; they don't add up to 1.

b. No; a probability cannot be negative.

c. No; a probability cannot be greater than 1.

d. Yes

9. a. $.1 + .6 = .7$

b. $.6 + .1 = .7$

10. a. $\Pr(E) = .05 + .25 = .30$
$\Pr(F) = .05 + .63 + .01 = .69$

b. $E' = \{s_3,\ s_4,\ s_5,\ s_6\};$
$\Pr(E') = .05 + .01 + .63 + .01 = .70$

c. $E \cap F = \varnothing;\ \Pr(E \cap F) = 0$

d. $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F) = .30 + .69 - 0 = .99$

11. a. $\frac{10}{10+1} = \frac{10}{11}$

b. $\frac{1}{1+2} = \frac{1}{3}$

c. $\frac{4}{4+5} = \frac{4}{9}$

12. $a = 1$ and $a + b = 6$, so $b = 5$.
Thus the odds are 1 to 5.

13. $.09 = \frac{9}{100}$
Thus $a = 9$ and $a + b = 100$, so $b = 91$. The odds are 9 to 91.

14. $\frac{16}{16+9} = \frac{16}{25}$

15. Win: $\frac{11}{11+7} = \frac{11}{18}$
Lose: $\frac{7}{11+7} = \frac{7}{18}$

16. a. $1 - (.18 + .23 + .31) = .28$

b. $.18 + .31 = .49$

c. $.18 + .23 + .28 = .69$

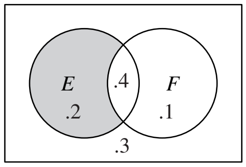
d. $\Pr(\text{Sue loses}) = .18 + .31 + .28 = .77 = \frac{77}{100};$
$a = 77$ and $a + b = 100$, so $b = 23$.
The odds are 77 to 23.

e. $\Pr(\text{a girl wins}) = .23 + .28 = .51 = \frac{51}{100};$
$a = 51$ and $a + b = 100$, so $b = 49$.
The odds are 51 to 49.

f. $\Pr(\text{Sam wins}) = .18 = \frac{18}{100} = \frac{9}{50};$
$a = 9$ and $a + b = 50$, so $b = 41$.
The odds are 9 to 41.

17. a. $\Pr(E \cup F) = \Pr(E) + \Pr(E) - \Pr(E \cap F) = .7$

b.



$\Pr(E \cap F') = \Pr(E) - \Pr(E \cap F) = .2$

18. a. $\Pr(E \cap F) = \Pr(E) - \Pr(E \cap F') = .1$

b. $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$
$= .8$

19. a. .20 + .25 + .25 = .7

b. .70 × 10,000 = 7000

20. Pr(*S*) = 1

21.

Number of Colleges Applied to	Probability
1	.20
2	Pr(2) = Pr(2 or less) – Pr(1) = .33 – .20 = .13
3	Pr(3) = Pr(3 or less) – Pr(2 or less) = .49 – .33 = .16
4	Pr(4) = Pr(4 or less) – Pr(3 or less) = .66 – .49 = .17
5–20	Pr(5 –20) = Pr(20 or less) – Pr(4 or less) = 1 – .66 = .34

22. $\Pr(3 \text{ or more}) = \Pr(3) + \Pr(4) + \Pr(5-20)$
$= .16 + .17 + .34 = .67$

23. a. Pr(20–34) = .15
Pr(35–49) = Pr(20–49) – Pr(20–34) = .55
Pr(50–64) = Pr(20–64) – Pr(20–49) = .20
Pr(65–79) = Pr(20–79) – Pr(20–64) = .10

b. Pr(50–79) = Pr(20–79) – Pr(20–49) = .30

24. a. $\dfrac{1600}{1200+1570+1600+1520+1480}$
$= \dfrac{1600}{7370}$
$= \dfrac{160}{737}$

b. Pr(house is less than 7 years old)
$= \dfrac{1200+1570+1600}{1200+1570+1600+1520+1480}$
$= \dfrac{4370}{7370}$
$= \dfrac{437}{737};$
a = 437 and *a* + *b* = 737, so *b* = 300. The odds are 437 to 300.

25. a. Some categories are left out—people who use a computer for both school and work, for example.

b. 100% – 17% = 83%

26. $\frac{.7}{1-.7} = \frac{.7}{.3} = \frac{7}{3}$
The odds in favor of a major earthquake are 7 to 3.

Exercises 6.4

1. a. There are 3 × 3 × 3 = 27 combinations of class selections.
$\frac{3}{27} = \frac{1}{9}$

b. $\frac{3 \times 2 \times 1}{27} = \frac{2}{9}$

2. The number of ways that Michael can be assigned to the inside lane and Christopher to the outside lane is 1 · 5 · 4 · 3 · 2 · 1 · 1 = 120.
The total number of possible assignments is 7! = 5040.
The probability in question is $\frac{120}{5040} = \frac{1}{42} \approx .024$

3. a. $\frac{7}{13}$

b. $\frac{6}{13}$

c. $\Pr(\{3, 6, 9, 12\}) = \frac{4}{13}$

d. $\Pr(\{1, 3, 5, 6, 7, 9, 11, 12, 13\}) = \frac{9}{13}$

4. a. $\frac{6^5}{13^5} = \left(\frac{6}{13}\right)^5 = \frac{7776}{371,293}$

b. $\frac{7^5}{13^5} = \left(\frac{7}{13}\right)^5 = \frac{16,807}{371,293}$

c. $1 - \left(\frac{6}{13}\right)^5 = \frac{363,517}{371,293}$

5. a. $\frac{C(6, 5)}{C(13, 5)} = \frac{6}{1287} = \frac{2}{429}$

b. $\frac{C(7, 5)}{C(13, 5)} = \frac{21}{1287} = \frac{7}{429}$

c. $1 - \frac{2}{429} = \frac{427}{429}$

6. .45(40) = 18 red balls

7. $\frac{C(6, 2) \times C(5, 2)}{C(11, 4)} = \frac{15 \times 10}{330} = \frac{5}{11}$

8. $1 - \frac{C(5, 3)}{C(10, 3)} = 1 - \frac{10}{120} = \frac{11}{12}$

9. Ways for exactly 2 to agree: $C(5, 2) \times C(4, 1)$
Ways for all 3 to agree: $C(5, 3)$
$\frac{C(5, 2) \times C(4, 1) + C(5, 3)}{C(9, 3)} = \frac{10 \times 4 + 10}{84} = \frac{25}{42}$

10. $1 - \frac{C(4, 2)}{C(9, 2)} = 1 - \frac{6}{36} = \frac{5}{6}$

11. Ways for no girls to be chosen: $C(12, 7)$
Ways for exactly 1 girl to be chosen:
$C(12, 6) \times C(10, 1)$
$1 - \frac{C(12, 7) + C(12, 6) \times C(10, 1)}{C(22, 7)}$
$= 1 - \frac{792 + 924 \times 10}{170,544} = \frac{16}{17}$

12. $\frac{C(10, 7)}{C(22, 7)} = \frac{5}{7106}$

13. Ways for no girl to be chosen: $C(12, 7)$
Ways for exactly 1 girl to be chosen: $C(12, 6) \times C(10, 1)$
Ways for exactly 2 girls to be chosen: $C(12, 5) \times C(10, 2)$
Ways for exactly 3 girls to be chosen: $C(12, 4) \times C(10, 3)$

$$\frac{C(12, 7) + C(12, 6) \times C(10, 1) + C(12, 5) \times C(10, 2) + C(12, 4) \times C(10, 3)}{C(22, 7)}$$
$$= \frac{792 + 9240 + 35,640 + 59,400}{170,544}$$
$$= \frac{199}{323}$$

14. $\frac{12 \cdot 11 \cdot 10}{22 \cdot 21 \cdot 20} = \frac{1}{7}$

15. $1 - \frac{30 \times 29 \times 28 \times 27}{30^4} = \frac{47}{250}$

16. $\frac{26 \times 25 \times 24 \times 23 \times 22}{26^5} = \frac{18,975}{28,561}$

17. $1 - \frac{6 \cdot 6}{7 \cdot 7} = \frac{13}{49}$

18. $1 - \frac{6 \times 5 \times 4 \times 3}{6^4} = \frac{13}{18}$

19. **a.** $.2 + .15 - .1 = .25$

b. $1 - .25 = .75$

c. $1 - .2 = .8$

20. $\Pr(\text{exactly 4 heads}) = \frac{C(7, 4)}{2^7} = \frac{35}{128}$

21. $\Pr(F) = 1 - \Pr(F') = .4$
$\Pr(E \cap F) = \Pr(E) + \Pr(F) - \Pr(E \cup F) = .3 + .4 - .7 = 0$

22. $\Pr(\text{I or II}) = \Pr(\text{I}) + \Pr(\text{II}) - \Pr(\text{I and II}) = .32$
Pr(neither I nor II) = 1 – .32 = .68

23. $\Pr(\text{second sock matches first}) = \frac{12 \cdot 1}{12 \cdot 11} = \frac{1}{11}$

24. $\frac{C(10, 4) \times C(5, 2)}{C(15, 6)} = \frac{60}{143}$

25. There are 5! = 120 ways to arrange the family.
First determine where the parents will stand. There are two ways with the man at one end. If the man doesn't stand at one end, there are $3 \cdot 2 = 6$ ways for the couple to stand together. Next determine the order of the children.
There are 3! = 6 ways to order the children.
Thus there are $(2 + 6) \cdot 6 = 48$ ways to stand with the parents together.
The probability is $\frac{48}{120} = \frac{2}{5}$.

26. .0019654 + .0039246 – .0000154 = .0058746

27. The tourist must travel 8 blocks of which 3 are south. Thus he has $C(8, 3) = 56$ ways to get to B from A.

a. To get from A to B through C there are $C(3, 1) \cdot C(5, 2) = 30$ ways.
The probability is $\frac{30}{56} = \frac{15}{28}$.

b. To get from A to B through D there are $C(5, 1) \cdot C(3, 2) = 15$ ways.
The probability is $\frac{15}{56}$.

c. To get from A to B through C and D there are
$C(3, 1) \cdot C(2, 0) \cdot C(3, 2) = 9$ ways.
The probability is $\frac{9}{56}$.

d. The number of ways to get from A to B through C or D is $30 + 15 - 9 = 36$.
The probability is $\frac{36}{56} = \frac{9}{14}$.

28. $\Pr(4 \text{ girls}) + \Pr(3 \text{ girls}) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

29. $1 - \frac{C(6, 3)}{C(10, 3)} = 1 - \frac{20}{120} = \frac{5}{6}$

30. $\frac{C(6, 3)}{2^6} = \frac{5}{16}$

31. $1 - \frac{C(8, 3)}{C(9, 3)} = 1 - \frac{2}{3} = \frac{1}{3}$

32. $\frac{C(4, 2)}{C(8, 2)} = \frac{3}{14}$

33. $\frac{2}{C(40, 6)} = \frac{1}{1{,}919{,}190}$

34. Many people think multiples of 7 are lucky, and some might also pick 13 just to prove they're not superstitious! To avoid sharing, avoid "lucky" numbers.

35. The total number of ways to select 5 senators is
$C(100, 5) = \frac{100!}{95!5!} = 75{,}287{,}520.$
The total number of ways to select senators from different states is
$\frac{100 \cdot 98 \cdot 96 \cdot 94 \cdot 92}{5!} = 67{,}800{,}320.$
Pr(no two members from same state)
$= \frac{67{,}800{,}320}{75{,}287{,}520}$
$\approx .90055$

36. 1-member subcommittee:
$\Pr(\text{winning}) = \frac{3}{9} = \frac{1}{3} \approx .33$
3-member subcommittee:
$$\Pr(\text{winning}) = \frac{C(3, 3) + C(3, 2) \times C(6, 1)}{C(9, 3)} = \frac{1 + 3 \times 6}{84} = \frac{19}{84} \approx .23$$
5-member subcommittee:
$$\Pr(\text{winning}) = \frac{C(3, 3) \times C(6, 2)}{C(9, 5)} = \frac{1 \times 15}{126} = \frac{5}{42} \approx .12$$
7-member subcommittee: Pr(winning) = 0

37. Pr(at least one birthday on June 13)
$= 1 - \left(\frac{364}{365}\right)^{25}$
$\approx .066$
Because in Table 1 no particular date is being matched. Any two (or more) identical birthdays count as a success.

38. There are 6! or 120 ways to arrange 5 letters. In 36 of these, the 3 E's will be adjacent.
$\Pr(\text{all } E\text{'s adjacent}) = \frac{36}{120} = \frac{3}{10}$

39. a. $\Pr(\text{Mary and Laura}) = \frac{C(26, 8)}{C(28, 10)} = \frac{1{,}562{,}275}{13{,}123{,}110} \approx .119$

b. $\Pr(\text{Mary and Laura}) = \dfrac{C(16, 5)\cdot C(10, 3)}{C(16, 5)\cdot C(12, 5)} = \dfrac{C(10, 3)}{C(12, 5)} = \dfrac{120}{792} \approx .152$

40. Pr(at least one correct) = 1 – Pr(none correct)
$= 1 - \dfrac{4\cdot 4\cdot 3}{5\cdot 5\cdot 4} = 1 - \dfrac{48}{100} = \dfrac{52}{100} = .52$

41. Pr(at least one winner) = 1 – Pr(no winners)
$= 1 - \dfrac{3\cdot 3\cdot 2}{4\cdot 4\cdot 3} = 1 - \dfrac{18}{48} = .625$

42. This decreases the chances because now the writer is not guaranteed to even have one team from each division.

43. **a.** $\left(\dfrac{1}{49}\right)^5\left(\dfrac{1}{42}\right) = \dfrac{1}{11{,}863{,}960{,}458}$
The odds are 1:11,863,960,457.

b. $\left(\dfrac{1}{49}\right)^5\left(\dfrac{1}{42}\right) = \dfrac{1}{11{,}863{,}960{,}458}$

44. Pr(first one is red) · Pr(second one is red) = .1
Let r = number of red balls.

$$\frac{r}{40}\cdot\frac{r-1}{39} = .1$$
$$r(r-1) = (40)(39)(.1)$$
$$r^2 - r - 156 = 0$$
$$(r+12)(r-13) = 0$$
$$r = 13$$

45. $1 - \dfrac{52\times 51\times 50\times 49\times 48}{52^5} = .1797$

$1 - \dfrac{52\times 51\times 50\times \cdots \times (52-n+1)}{52^n}$ exceeds .5 starting with $n = 9$.

46. **a.** $\dfrac{1}{6}\times 24 = 4$

b. $\dfrac{C(24, 4)\times 5^{20}}{6^{24}} \approx .2139$

47. $1 - \dfrac{16\times 15\times 14\times 13\times 12}{16^5} \approx .5001$

48. $1 - \dfrac{N\times(N-1)\times(N-2)\times\cdots\times(N-20+1)}{N^{20}}$
dips below .5 starting with $N = 281$.

Exercises 6.5

1. $\Pr(E|F) = \dfrac{.1}{.3} = \dfrac{1}{3},\ \Pr(F|E) = \dfrac{.1}{.5} = \dfrac{1}{5}$

2. No; $\Pr(E|F) \neq \Pr(E)$

3. $\Pr(E|F') = \dfrac{\Pr(E\cap F')}{\Pr(F')} = \dfrac{P(E) - \Pr(E\cap F)}{1-\Pr(F)} = \dfrac{.4}{.7} = \dfrac{4}{7}$

4. From Exercise 3, $\Pr(E|F') = \dfrac{4}{7}$.

$\Pr(E'|F') = 1 - \Pr(E|F') = \dfrac{3}{7}$

5. No;
$\Pr(\text{has cancer}|\text{works for Ajax}) \neq \Pr(\text{has cancer})$

6.
$\Pr(\text{college grad}|\text{high earner})$
$= \dfrac{\Pr(\text{college grad \& high earner})}{\Pr(\text{high earner})} = \dfrac{.10}{.25} = .40$

7. **a.** .80 × .75 × .60 = .36

b. .36 + (1 – .80)(.75)(.60) + (.80)(1 – .75)(.60) + (.80)(.75)(1 – .60) = .81

8. $(1 - .0005)^{50} \approx .9753$

9. $(1-.01)^5(1-.02)^5(1-.025)^3=(.99)^5(.98)^5(.975)^3\approx .7967$

10. Pr(all burn out) = $.01^N$ drops below (1 – .99999) = .00001 for N = 3.

11. $\Pr(E|F)=\dfrac{\frac{1}{4}}{\frac{1}{3}}=\dfrac{3}{4}$

$\Pr(F|E)=\dfrac{\frac{1}{4}}{\frac{1}{2}}=\dfrac{1}{2}$

12. **a.** $\Pr(E\cap F)=\Pr(E)+\Pr(F)-\Pr(E\cup F)=.3+.6-.7=.2$

b. $\dfrac{\Pr(E\cap F)}{\Pr(F)}=\dfrac{.2}{.6}=\dfrac{1}{3}$

c. $\dfrac{\Pr(E\cap F)}{\Pr(E)}=\dfrac{.2}{.3}=\dfrac{2}{3}$

d. $\Pr(F)-\Pr(E\cap F)=.6-.2=.4$

e. $\dfrac{\Pr(E'\cap F)}{\Pr(F)}=\dfrac{.4}{.6}=\dfrac{2}{3}$

13. **a.** 1 – Pr(Dem. or favors) = 1 – [Pr(Dem.) + Pr(favors) – Pr(Dem. and favors)] = .4

b. $\dfrac{\text{Pr(Dem. and favors)}}{\text{Pr(Dem.)}}=.6$

c. $\dfrac{\text{Pr(Dem. and favors)}}{\text{Pr(favors)}}=.75$

14. **a.** 1 – Pr(uppercl. or doesn't attend) = 1 – [Pr(uppercl.) + Pr(doesn't attend) – Pr(uppercl. and doesn't attend)]
1 – (.7 + .5 – .4) = .2

b. $\dfrac{\text{Pr(first - year and attends)}}{\text{Pr(first - year)}}=\dfrac{.2}{.3}=\dfrac{2}{3}$

c. $\dfrac{\text{Pr(first - year and attends)}}{\text{Pr(attends)}}=\dfrac{.2}{.5}=\dfrac{2}{5}$

15. $\dfrac{\Pr(\{\text{HHH}\})}{\Pr(\{\text{HHH, HHT, HTH, THH}\})}=\dfrac{1}{4}$

16. $\dfrac{\text{[number of outcomes that two are white]}}{\text{[number of outcomes that at least 1 is white]}}=\dfrac{C(2,\,2)}{C(2,\,2)+C(3,\,1)\times C(2,\,1)}=\dfrac{1}{1+6}=\dfrac{1}{7}$

17. 1 – Pr(Neither A nor B lives 15 more years) = 1 – (1 – .8)(1 – .7) = .94

18. No; the probability of the second clearly changes if the first is given to have occurred.
Let E = "the sample contains at least one white ball" and F = "the sample contains balls of both colors."
Then $\Pr(F)=\dfrac{C(2,\,2)+C(3,\,2)}{C(5,\,2)}=\dfrac{2}{5}$ and $\Pr(F|E)=\dfrac{1}{7}$.

19. $(1-.6)(1-.6)=.16$

20. Pr(tuna on each of three) = .15(.15)(.15) ≈ .0034
Pr(tuna on at least one) = 1 − Pr(no tuna) = 1 − .85(.85)(.85) ≈ .386

21. $1-(1-.005)^2=.009975$

22. $\left(\frac{1}{2}\right)^{10}=\frac{1}{1024}$

23. $(.7)^4=.2401$

24. $\Pr(A\cap B)=\Pr(A)\cdot\Pr(B|A)=.10$
$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)=.60$
$\Pr(A'\cap B)=\Pr(B)-\Pr(A\cap B)=.20$
$\Pr\left(A|B\right)=\frac{\Pr(A\cap B)}{\Pr(B)}=\frac{.10}{.30}=\frac{1}{3}$

25. $\Pr(E\cap F)=\Pr(E)\cdot\Pr(F)$ Definition of "independent"
$\Pr(E)+\Pr(F)-\Pr(E\cup F)=\Pr(E)\cdot\Pr(F)$ Substitution
$[1-\Pr(E')]+[1-\Pr(F')]-[1-\Pr(E'\cap F')]=[1-\Pr(E')]\cdot[1-\Pr(F')]$ Substitution
$\Pr(E'\cap F')=\Pr(E')\cdot\Pr(F')$ Simplification

26. From Exercise 31, since E and F are independent, so are E' and F'.
$\Pr(E\cup F)=1-P(E'\cap F')=1-\Pr(E')\cdot\Pr(F')$
Also recall that $E'\cap F'=(E\cup F)'$.

27. 0 points: 1 − .6 = .4
1 point: .6 × .4 = .24
2 points: .6 × .6 = .36

28. **a.** $\Pr\left(E'|F\right)=\frac{\Pr(E'\cap F)}{\Pr(F)}$
$=\frac{\Pr(F)-\Pr(E\cap F)}{\Pr(F)}=\frac{\Pr(F)}{\Pr(F)}-\frac{\Pr(E\cap F)}{\Pr(F)}$
$=1-\Pr\left(E|F\right)$

b. Possible answer: E = roll a 6;
F = roll an even number with a die
$\Pr\left(E|F'\right)=0\neq 1$

29. $\Pr\left(E\cup F|G\right)=\frac{\Pr[(E\cup F)\cap G]}{\Pr(G)}$
$=\frac{\Pr[(E\cap G)\cup(F\cap G)]}{\Pr(G)}$
$=\frac{\Pr(E\cap G)+\Pr(F\cap G)-\Pr[(E\cap G)\cap(F\cap G)]}{\Pr(G)}$
$=\frac{\Pr(E\cap G)}{\Pr(G)}+\frac{\Pr(F\cap G)}{\Pr(G)}-\frac{\Pr[(E\cap F)\cap G]}{\Pr(G)}$
$=\Pr(E|G)+\Pr(F|G)-\Pr(E\cap F|G)$

30. Pr(ever married|aged 25–29) = .548

31. a. $\Pr(\text{death}|A) = \frac{12{,}000}{120{,}000} = \frac{1}{10}$

b. $1000 \times \frac{1}{10} = 100$ per 1000

c. $\Pr(\text{death}|B) = \frac{4500}{90{,}000} = \frac{1}{20}$

32. Let A = "first test correctly identifies blood type" and
B = "second test correctly identifies blood type."

a. $\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$
$= .7 + .8 - .9$
$= .6$

b. $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{.6}{.7} = \frac{6}{7}$

c. $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{.6}{.8} = \frac{6}{8} = \frac{3}{4}$

d. No; $\Pr(A \cap B) \neq \Pr(A) \cdot \Pr(B)$

33. a. $\Pr(B) = 80\% \times (100\% - 65\%) = .28$

b. $\Pr(C) = 20\% \times (100\% - 65\%) = .07$

c. $\Pr(H) = (65\% \times 75\%) + (28\% \times 40\%) + (7\% \times 10\%) = .6065$

34. a. $\frac{250-(160+50)}{250} = \frac{40}{250} = \frac{4}{25} = .16$

b. $\frac{90+40}{250} = \frac{13}{25} = .52$

c. $\frac{250-(90+50)}{250} = \frac{110}{250} = \frac{11}{25} = .44$

d. $\frac{250-160}{250} = \frac{9}{25} = .36$

e. $\frac{160-90}{110} = \frac{7}{11} \approx .64$

f. $\frac{160-90}{160} = \frac{7}{16} \approx .44$

g. $\frac{160}{250-50} = \frac{4}{5} = .80$

h. $\frac{40}{250-160} = \frac{4}{9} \approx .44$

i. $\frac{50}{90+50} = \frac{5}{14} \approx .36$

35. a. $\frac{250-(120+140-50)}{250} = \frac{40}{250} = \frac{4}{25} = .16$

b. $\frac{(120-50)+(140-50)}{250} = \frac{16}{25} = .64$

c. $\frac{140-50}{250} = \frac{9}{25} = .36$

d. $\frac{50}{120} = \frac{5}{12} \approx .42$

e. $\frac{120-50}{250-140} = \frac{7}{11} \approx .64$

f. $\frac{40}{250-120} = \frac{4}{13} \approx .31$

36. Total students $= 130 + 460 + 210 + 100 + \cdots + 50$
$= 2700$

a. $\frac{200+300+50}{2700} \approx .2037$

b. $\frac{130+100+80+200}{2700} \approx .1889$

c. $\frac{460}{130+460+210} = .5750$

d. $\frac{210}{210+150+100+50} \approx .4118$

e. $\frac{100+50}{210+150+100+50} \approx .2941$

f. $\frac{100+500+80+420}{100+500+150+80+420+100} \approx .8148$

37. Total voters $= 400 + 700 + \cdots + 200 = 2500$

a. $\frac{400+600}{2500} = .40$

b. $\frac{400+700+300}{2500} = .56$

c. $\frac{200}{600+300+200} \approx .18$

d. $\dfrac{700}{700+300} = .70$

e. $\dfrac{600+300}{400+700+600+300} = .45$

f. $\dfrac{400+600}{400+600+300+200} \approx .67$

38. Total population = 1000 + 1200 + ⋯ + 10,000
= 147,250

a. $\dfrac{100,000+35,000+10,000}{147,250} \approx .985$

b. $\dfrac{1000+1200+50}{147,250} \approx .015$

c. $\dfrac{1200}{1200+35,000} \approx .033$

d. $\dfrac{1000+50}{1000+100,000+50+10,000} \approx .009$

e. No; Pr(has virus|Afro-American)
≠ Pr(has virus)

39. a. Total active military duty = 76,140 + 53,922 + ⋯ + 280,410 = 1,272,171

$$\Pr(E) = \frac{397{,}703+321{,}762+55{,}264+280{,}410}{1{,}272{,}171} \approx .8294$$

$$\Pr(N) = \frac{53{,}922+321{,}762}{1{,}272{,}171} \approx .2953$$

$$\Pr(E\cap N) = \frac{321{,}762}{1{,}272{,}171} \approx .2529$$

$$\Pr\left(E|N\right) = \frac{\Pr(E\cap N)}{\Pr(N)} = \frac{.2529}{.2953} \approx .8564$$

$$\Pr\left(N|E\right) = \frac{\Pr(E\cap N)}{\Pr(E)} = \frac{.2529}{.8294} \approx .3049$$

b. No, events E and N are not independent because $\Pr(E|N) \neq \Pr(E)$.
Note: the fact that $\Pr(N|E) \neq \Pr(N)$ could also be used.

40. a. Pr(at least one hit) = 1 – Pr(no hits)
$= 1 - (.7)^4 = .7599$

b. Pr(at least one hit in each of first 10 games) = $(.7599)^{10} \approx .0642$

c. Pr(at least one has 10-game hitting streak) = 1 – Pr(no 10-game hitting streaks)
$= 1 - (1 - .0642)^{20}$
$\approx .7348$

41. Less than 70 and divisible by 7: 7, 14, 21, 28, 35, 42, 49, 56, 63. Less than 70 and divisible by 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69. In both groups: 21, 42, 63.

Pr(both choose 21) = Pr(both choose 42) = Pr(both choose 63) $=\frac{1}{9}\times\frac{1}{23}=\frac{1}{207}$.

$\Pr(\text{choose same})=\frac{1}{207}+\frac{1}{207}+\frac{1}{207}=\frac{1}{69}$

42. 0; the smallest positive integer divisible by 7 and 11 is 77.

43. $\Pr(\text{all red})=\frac{C(6,\ 3)}{C(13,\ 3)}=\frac{10}{143}$

$\Pr(\text{all blue})=\frac{C(4,\ 3)}{C(13,\ 3)}=\frac{2}{143}$; $\Pr(\text{all white})=\frac{C(3,\ 3)}{C(13,\ 3)}=\frac{1}{286}$

Pr(all same color) = Pr(all red) + Pr(all blue) + Pr(all white) $=\frac{10}{143}+\frac{2}{143}+\frac{1}{286}=\frac{25}{286}\approx .0874$

44. Pr(odd followed by even) + Pr(even followed by odd) $=\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{2}$

45. $\Pr(\text{both roast beef})=\Pr(\text{both ham})=\frac{C(2,\ 2)}{C(4,\ 2)}=\frac{1}{6}$

Pr(both roast beef) + Pr(both ham) $=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

46. Pr(both odd) + Pr(both even) $=\frac{5}{10}\cdot\frac{4}{9}+\frac{5}{10}\cdot\frac{4}{9}=\frac{4}{9}$

47. Pr(all white|one white) = $\frac{\text{Pr(all white and one white)}}{\text{Pr(one white)}}=\frac{\frac{8}{13}\cdot\frac{7}{12}\cdot\frac{6}{11}\cdot\frac{5}{10}}{1-\frac{5}{13}\cdot\frac{4}{12}\cdot\frac{3}{11}\cdot\frac{2}{10}}\approx .0986$

48. 1 – Pr(no match) $=1-\frac{52\times51\times50\times49\times48\times47\times46\times45}{52^8}\approx .4324$

49. If E and F are mutually exclusive then $\Pr(E\cap F)=0$.

Thus, $\Pr(E\mid F)=\frac{\Pr(E\cap F)}{P(F)}=\frac{0}{\Pr(F)}=0.$

Since $\Pr(E)\neq 0$, $\Pr(E\mid F)\neq\Pr(E)$.

Therefore, E and F are not independent.

50. From Exercise 49, we see that any mutually exclusive events with nonzero probabilities are *not* independent. Consider any event E and its complement E', which are mutually exclusive, where $\Pr(E)\neq 0$ and $\Pr(E')\neq 0$.

51. The sample space for 2 children is {*BB*, *BG*, *GB*, *GG*}. The event "at least one boy" is {*BB*, *BG*, *GB*). The event "older is boy" is {*BB*, *BG*}.

Pr(2 boys|at least one boy) $=\frac{\Pr(\{BB\})}{\Pr(\{BB,\ BG,\ GB\})}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$

Pr(2 boys|older is boy) $=\frac{\Pr(\{BB\})}{\Pr(\{BB,\ BG\})}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

No, the man has a greater chance of having two boys.

52. $\Pr(W) = \frac{66{,}071}{141{,}814} \approx .4659$

$\Pr(U) = \frac{6742}{141{,}814} \approx .0475$

$\Pr(W \cap U) = \frac{3079}{141{,}814} \approx .0217$

Gender and unemployment status are not independent because $\Pr(W \cap U) \neq \Pr(W) \cdot \Pr(U)$.

53. a. $\frac{313 - 260}{313 + 8249} \approx .0062$

b. $\frac{260}{313} \approx .8307$

c. $\frac{260 + 14}{313 + 8249} \approx .0320$

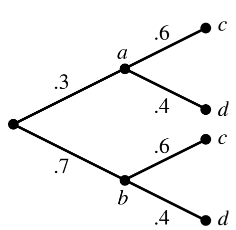
54. a. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \approx 3.69 \times 10^{-6}$

b. $\frac{2}{52} \cdot \frac{1}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \approx 6.16 \times 10^{-7}$

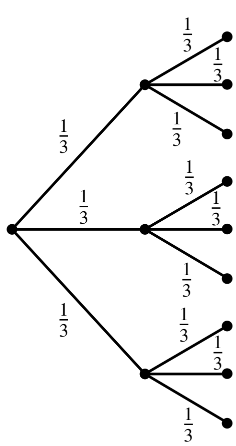
c. (a)

Exercises 6.6

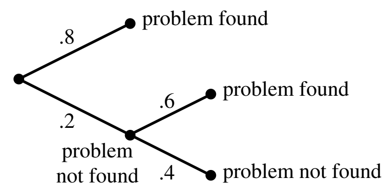
1.



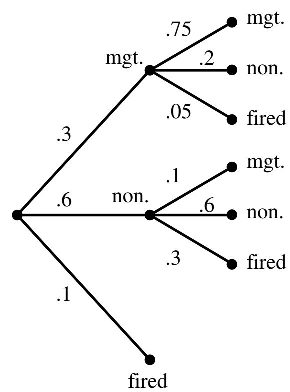
2.



3.



4.



5. $(1 - .8)(1 - .6) = .08$

6. $.30 \times .75 + .60 \times .10 = .285$

7. $.10 + .30 \times .05 + .60 \times .30 = .295$

8. $.30 \times (.20 + .05) = .075$

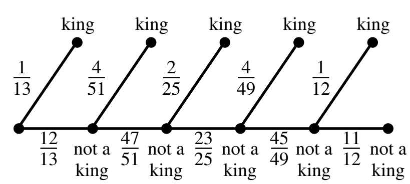
9. Pr(white, then red) + Pr(red, then red)

$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{4} = \frac{7}{12}$

10. Pr(6 on die) $= \frac{40}{52} \times \frac{1}{6} = \frac{5}{39}$;

Pr(head on coin) $= \frac{12}{52} \times \frac{1}{2} = \frac{3}{26}$

11.



Pr(king on 1st draw) + Pr(king on 2nd draw) + Pr(king on 3rd draw) = 1 – Pr(not a king on 3rd draw)

$= 1 - \frac{12}{13} \times \frac{47}{51} \times \frac{23}{25} = 1 - \frac{4324}{5525} = \frac{1201}{5525} \approx .22$