MTH 3001 – Fall 2001

Extra Problem Set

1. A graph K_n has n vertices, and every two vertices are joined by an edge.

   (a) Draw K_n for $n = 1, \ldots, 6$.

   (b) For which values of n is K_n a cycle? For which values of n is K_n a tree?

   (c) How many edges does K_n have?

   (d) How many edges does a spanning tree for K_n have?

2. Let $\delta(G) := \min\{\deg(v) : v \in V\}$ be the *minimum degree* of a graph $G = (V, E)$. Assume $\delta(G) \geq 2$. Show that:

   (a) G contains a path of length $\delta(G)$.

   (b) G contains a cycle of length at least $\delta(G) + 1$.

3. Let $\chi(G) := \min\{k : G \text{ is } k\text{-colorable}\}$ be the *chromatic number* of a graph $G = (V, E)$. Show that:

$$\chi(G) \leq \frac{1}{2} + \sqrt{2|E| + \frac{1}{4}}\,.$$

4. In how many ways can three distinct numbers be chosen from the set $\{1, 2, 3, \ldots, 200\}$ so that their sum is even?

5. Find

$$\sum_{k=0}^{n} \binom{n}{k} 3^k.$$