

Please, justify your answers.

1. Find the matrix A of the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ given by

$$y_1 = -x_1 + 3x_2 + 2x_3$$

$$y_2 = -5x_2 + 2x_3$$

$$y_3 = 7x_1 + x_2 - 4x_3$$

$$y_4 = x_2$$

2. Consider the linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$, where

$$T\vec{e}_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad T\vec{e}_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad T\vec{e}_3 = \begin{bmatrix} 0 \\ 5 \end{bmatrix}, \quad T\vec{e}_4 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

- (a) Find the matrix A of T .

(b) Compute $T \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

3. Let $A = [1 \ 2]$, $B = \begin{bmatrix} 5 & -2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$, $C = [0 \ 0 \ 1]$,

Decide whether the following products are defined or not. If they are, compute them:
 AB, BA, AC, CA, BC, CB .

4. For which choices of the constant k is the following matrix invertible? $A = \begin{bmatrix} k & k & 1 \\ 0 & 4 & 0 \\ 1 & 1 & k \end{bmatrix}$

5. Find the matrix A of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with

$$T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and } T \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

6. Use Gaussian elimination to find the inverse of following matrix. Indicate for each step which

operation you use. $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$