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February 5, 2009

Please, justify your answers.

1. Consider the linear transformation given by multiplication by the matrix A , as $A\vec{x}$ for each vector $\vec{x} \in \mathbb{R}^n$.

$$A = \begin{bmatrix} -1 & 3 & 2 & 0 \\ 1 & -3 & -1 & 0 \\ 1 & -3 & -4 & 0 \end{bmatrix}$$

$$\mathbb{R}^n \xrightarrow{A} \mathbb{R}^m$$

- (a) $n =$ _____
- (b) $m =$ _____
- (c) What is the dimension of the Domain of this transformation? _____
- (d) What is the dimension of the Codomain of this transformation? _____
- (e) Find the $rref A$.
- (f) Find the image of A , ImA
- (g) Find the kernel of A , $KerA$
2. Find the matrix of rotation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ through the angle $\alpha = \pi/3$.
3. Find the matrix of the projection onto the $\{\vec{e}_2, \vec{e}_3\}$ plane in \mathbb{R}^3 .
4. Find the matrix of the dilation by a factor of $-1/3$ in \mathbb{R}^2 .

5. Find the matrix of the reflection through \vec{e}_1 -line $\mathbb{R}^2 \rightarrow \mathbb{R}^2$.

6. Consider the vector $\vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(a) Find the length $|\vec{v}|$ of \vec{v} .

(b) Find the unit vector $|\vec{u}|$ which determines the same line as the vector \vec{v} , i.e. $L_{\vec{u}} = L_{\vec{v}}$.

(c) Find the projection of the vector $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ onto the line $L_{\vec{v}}$.

(d) Find the projection of the vector $\vec{e}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ onto the line $L_{\vec{v}}$.

(e) Find the matrix A of the projection onto the line $L_{\vec{v}}$.

(f) (Extra credit!) Use the matrix A , from the previous step to find the matrix B of the reflection through the line $L_{\vec{v}}$.