S09 MTH 371 Linear Algebra **NEW PRACTICE QUIZ 3** Name:

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Please, justify your answers.

1. Consider the linear transformation given by multiplication by the matrix A, as $A\vec{x}$ for each vector $\vec{x} \in \mathbb{R}^n$.

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & -1 \\ 1 & -3 & -4 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\mathbb{R}^n \xrightarrow{A} \mathbb{R}^m}$$

   (a) n= _____

   (b) m= _____

   (c) What is the dimension of the Domain of this transformation? _____

   (d) What is the dimension of the Codomain of this transformation?_____

   (e) Find the $rrefA$.

   (f) Find the image of A, ImA

   (g) Find the kernel of A, $KerA$

2. Find the matrix of rotation $\mathbb{R}^2 \to \mathbb{R}^2$ through the angle $\alpha = \pi/6$.

3. Find the matrix of the projection onto the $\{\vec{e}_1, \vec{e}_3\}$ plane in $\mathbb{R}^3$.

4. Find the matrix of the dilation by a factor of 5 in $\mathbb{R}^2$.

5. Find the matrix of the reflection through $\vec{e}_2$ line $\mathbb{R}^2 \to \mathbb{R}^2$.

6. Consider the vector $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

   (a) Find the length $|\vec{v}|$ of $\vec{v}$.

   (b) Find the unit vector $|\vec{u}|$ which determines the same line as the vector $\vec{v}$, i.e. $L_{\vec{u}} = L_{\vec{v}}$.

   (c) Find the projection of the vector $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ onto the line $L_{\vec{v}}$.

   (d) Find the projection of the vector $\vec{e}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ onto the line $L_{\vec{v}}$.

   (e) Find the matrix A of the projection onto the line $L_{\vec{v}}$.

   (f) Use the matrix A, from the previous step to find the matrix B of the reflection through the line $L_{\vec{v}}$.