S09 MTH 371 Linear Algebra **QUIZ 4** Name: _____

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1.

(4pts) (a) Prove that the following vectors are linearly dependent.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 3 \\ -2 \\ 1 \\ -2 \end{bmatrix}.$$

(4pts) (a) Exhibit a linear dependence relation among them.

2. Let V be the subspace of $\mathbb{R}^3$ defined by the equations: $\begin{matrix} x_1 - 3x_2 + 2x_3 = 0 \\ 2x_1 - 6x_2 + x_3 = 0 \end{matrix}$.

(1pts) (a) Find a matrix A such that V is the kernel of A.

(4pts) (b) Express V as the span of a set of vectors.

(4pts) (c) Express V as the image of a matrix B.

3. (6pts) Find the set of all vectors $\vec{x} \in \mathbb{R}^3$ such that $\left(\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} \cdot \vec{x} \right) = 0$.

4. Consider the liner transformation given by the multiplication by matrix A as $A\vec{x}$.

The matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 12 & 18 & 9 & 0 \\ 11 & 22 & 33 & 14 & -5 \end{bmatrix}$ has row-reduced echelon form: $rrefA = \begin{bmatrix} 1 & 2 & 3 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(1pt) (a) Fill in $\mathbb{R}^{\square} \xrightarrow{A} \mathbb{R}^{\square}$.

(4pts) (b) Find ImA and $dimImA$.

(7pts) (c) Find a basis for $KerA$ and $dimKerA$.

(1.5pts each) 5. True-False; Always-Sometimes-Never; Examples *(In each case give a short explanation.)*

T - F *If a 4×4 matrix A has $rankA = 3$ then $dimKerA = 1$.*

T - F *If the vectors $\vec{v}_1$ and $\vec{v}_2$ are in ImA, then the vector $5\vec{v}_1 - 3\vec{v}_2$ is in ImA.*

T - F *Consider a system of 5 equations in 5 variables. let A be the matrix of coefficients. If the system has ∞ many solutions, then $dimImA \leq 4$*

(A - S - N) *If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then $dimImA = 3$.*

(A - S - N) *Consider a system of 5 equations in 3 variables. let A be the matrix of coefficients. If the system has ∞ many solutions, then $rankA \leq 2$.*

(A - S - N) *Let A be a 3×3 matrix. If $detA = 2$ then $dimImA = 2$.*

EXAMPLE *Give an example of a matrix A with $rankA = 1$ and $dimKerA = 2$.*

EXAMPLE *Give an example of a 2×2 matrix A with $KerA = \vec{0}$.*

EXAMPLE *Give an example of a system of equations with at least one free variable.*