S09 MTH 371 Linear Algebra **NEW PRACTICE QUIZ 4** Name: ______________

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Please, justify your answers.

1. Let $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & -4 \\ -1 & 5 & -7 \end{bmatrix}$.

   Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

2. For which value(s) of the constant k do the vectors below form a basis of $\mathbb{R}^4$?

   $$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ k \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 3 \\ -2 \\ 1 \\ k \end{bmatrix}.$$

3. Consider the liner transformation given by the multiplication by matrix A as $A\vec{x}$.

   The matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 18 & 20 \end{bmatrix}$ has row-reduced echelon form: $rref A = \begin{bmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) Fill in $\mathbb{R}^{\square} \xrightarrow{A} \mathbb{R}^{\square}$.

   (b) Find the image of A, ImA.

   (c) Find a basis for ImA.

   (d) Find the dimension of ImA.

   (e) Find the kernel of A, $KerA$.

   (f) Find a basis for $KerA$.

   (g) Find the dimension of $KerA$.

4. Consider the 5x4 matrix $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \end{bmatrix}$. We are told the vector $\begin{bmatrix} -5 \\ 4 \\ -3 \\ 2 \end{bmatrix}$, is in the kernel of A. Write $\vec{v}_4$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

5. Let V be the subspace of $\mathbb{R}^3$ defined by the equation $3x_1 + 5x_2 - x_3 = 0$.

   (a) Express V as the kernel of a matrix A.

   (b) Express V as the image of a matrix B.

6. Let V be the subspace of $\mathbb{R}^3$ defined by the equations
   $x_1 + 2x_2 = 0$
   $3x_1 + 5x_2 - x_3 = 0$.

   (a) Express V as the kernel of a matrix A.

   (b) Express V as the image of a matrix B.

7. Find the set of all vectors $\vec{x} \in \mathbb{R}^3$ such that $\left(\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \cdot \vec{x} \right) = 0$.

8. Find the set of all vectors $\vec{x} \in \mathbb{R}^3$ such that $\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \vec{x} \right) = 0$ and $\left(\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \cdot \vec{x} \right) = 0$.

9. Examples

   (a) Give an example of a matrix A with $dimKerA = 2$ and $dimImA = 5$.
   (b) Give an example of a square matrix A with $dimKerA = 2$ and $dimImA = 5$.
   (c) Give an example of a matrix A with $dimKerA = 0$ and $dimImA = 5$.
   (d) Give an example of a matrix A with $dimKerA = 2$ and $dimImA = 0$.
   (e) Give an example of a square matrix A with $KerA = \mathbb{R}^2$.
   (f) Give an example of a square matrix A with $ImA = \mathbb{R}^2$.
   (g) Give an example of a square matrix A with $ImA \subset \mathbb{R}^3$ and $dimImA = 2$.
   (h) Give an example of an invertible matrix A.
   (i) Give an example of a non-invertible matrix A.
   (j) Give an example of a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$.

10. True-False

T - F If a 4×4 matrix A has $rankA = 3$ then $dimKerA = 3$.

T - F If a vector $\vec{v}$ is in $KerA$, then the vector $5\vec{v}$ is in $KerA$.

T - F Consider a system of 5 equations in 5 variables. let A be the matrix of coefficients. If the system has ∞ many solutions, then $dimImA = 5$

T - F Consider a system of 5 equations in 5 variables. let A be the matrix of coefficients. If the system has exactly one solution, then $dimKerA = 5$.

T - F If a 3×3 matrix A has $dimImA = 1$ then $dimKerA = 2$.

T - F If a 5×3 matrix A has $dimImA = 1$ then $dimKerA = 2$.

11. **A**lways-**S**ometimes-**N**ever

(A - S - N) If A is 3×3 matrix, then $dimImA \leq 3$.

(A - S - N) If A is 4×4 matrix, then $dimImA \leq 3$.

(A - S - N) If A is 5×5 matrix, then $dimImA = 3$.

(A - S - N) If A is 5×5 matrix, then $dimImA = 6$.

(A - S - N) If A is 5×2 matrix, then $dimImA = 6$.

(A - S - N) If A is 5×2 matrix, then $dimImA + dimKerA = 5$.

(A - S - N) If a system $A\vec{x} = \vec{0}$ has a non-leading variable, then $KerA$ has dimension at least 1.

(A - S - N) Let $A\vec{x} = \vec{0}$. Then #{leading variables} + #{non-leading variables}=#{columns of A}.

(A - S - N) If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then $dimImA =$ #{leading $\boxed{1}$} in the $rrefA$.

(A - S - N) If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then $dimImA =$ 3.

(A - S - N) If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then $dimImA =$ 2.

(A - S - N) If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then $dimImA \leq$ 2.