S09 MTH 371 Linear Algebra **QUIZ 5** Name: ____________________

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1. Let $V = span\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$. Find $V^{\perp}$.

2. Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   (a) Find the matrix of the orthogonal projection onto $V = span\{\vec{v}_1, \vec{v}_2\}$.

   (b) Find the orthogonal projection of $\vec{e}_1 \in \mathbb{R}^3$ onto the subspace V.

3. Let $A = \begin{bmatrix} 6 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$. Write the matrix A as the product $A = Q \cdot R$, where the columns of the matrix Q are ortho-normal and R is an upper triangular matrix.

4. **E**xamples; **T**rue-**F**alse; **A**lways-**S**ometimes-**N**ever *Some explanation is necessary!*

(a) Give an example of a matrix A with columns which form a basis for R^2 which is NOT orthonormal.

(b) Give an example of a set of vectors which are orthogonal but not orthonormal.

T - F If A is a 1×4 matrix then $A^T A$ is a 4×4 matrix.

T - F Consider a system of 5 equations in 5 variables. let A be the matrix of coefficients. If the system has exactly one solution, then $dim Ker A = 0$.

(A - S - N) If A is 1×5 matrix, then $rank(AA^T) \leq 1$.

(A - S - N) If A is 5×5 matrix, then $rank(A^T A) = 5$.

In the following problems, multiplication by matrix is always assumed to be $A\vec{x}$

(A - S - N) If multiplication by matrices A and B defines transformations $A : \mathbb{R}^2 \to \mathbb{R}^3$ and $B : \mathbb{R}^3 \to \mathbb{R}^5$ then multiplication by a matrix BA defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^5$.

(A - S - N) If multiplication by a matrix A defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then multiplication by AA^T defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$.