S09 MTH 371 Linear Algebra **NEW PRACTICE QUIZ 5** Name: ____________

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1. Apply the Gram-Schmidt process to the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and write the result in the form $A = Q \cdot R$.

2. Consider the subspace $V \subset \mathbb{R}^3$ which is spanned by the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$. Find an orthonormal basis for V.

3. Let $A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & -4 \\ -1 & 5 & -7 \end{bmatrix}$. Write the matrix A as the product $A = Q \cdot R$, where the columns of the matrix Q form an ortho-normal basis for $\mathbb{R}^3$ and $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ -1 & 5 \end{bmatrix}$. Write the matrix A as the product $A = Q \cdot R$, where the columns of the matrix Q are ortho-normal and R is an upper triangular matrix.
(Notice that both A and Q are 3×2 matrices and R is a square 2×2 matrix.)

5. Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix}$.

(a) Find the matrix of the orthogonal projection onto the line L in $\mathbb{R}^3$ spanned by $\vec{v}$.

(b) Find the orthogonal projection of $\vec{w}$ onto the line L.

6. Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix}$.

(a) Find the matrix of the orthogonal projection onto the subspace V in $\mathbb{R}^3$ spanned by $\vec{v}$ and $\vec{w}$.

(b) Find the orthogonal projection of $\vec{e}_1 \in \mathbb{R}^3$ onto the subspace V.

7. Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix}$.

(a) Let V be the subspace in $\mathbb{R}^3$ spanned by $\vec{v}$ and $\vec{w}$. Find the set of all vectors which are orthogonal to the subspace V.

(b) Find a basis for $V^{\perp}$.

8. Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix}$.

(a) Let V be the subspace in $\mathbb{R}^3$ spanned by $\vec{v}$ and $\vec{w}$. Find an orthogonal basis for the subspace V.

(b) Find an ortho-normal basis for $V^{\perp}$.

9. Let $V = span \left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$. Find $V^{\perp}$.

10. Let $V = span \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$. Find $V^{\perp}$.

11. Examples

    (a) Give an example of a matrix A with columns which form an orthonormal basis for R^2.

    (b) Give an example of a matrix A with columns which form a basis for R^2 which is NOT orthonormal.

    (c) Give an example of a set of orthonormal vectors in $\mathbb{R}^3$ which do not form an orthonormal basis for $\mathbb{R}^3$.

12. True-False

T - F If A is a 3×4 matrix then AA^T is a 3×3 matrix.

T - F If A is a 3×4 matrix then AA^T is a 4×3 matrix.

T - F If A is a 3×4 matrix then $A^T A$ is a 3×3 matrix.

T - F If A is a 3×4 matrix then A^T is a 4×3 matrix.

T - F Consider a system of 5 equations in 5 variables. Let A be the matrix of coefficients. If the system has ∞ many solutions, then $dim Im A = 5$

T - F Consider a system of 5 equations in 5 variables. let A be the matrix of coefficients. If the system has exactly one solution, then $dim Ker A = 0$.

13. **A**lways-**S**ometimes-**N**ever

(A - S - N) If A is 2×5 matrix, then $rank(AA^T) = 5$.

(A - S - N) If A is 4×4 matrix, then A^T is 4×4 matrix.

(A - S - N) If A is 5×1 matrix, then AA^T is 5×5.

(A - S - N) If multiplication by a matrix A, as $A\vec{x}$, defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ and multiplication by a matrix B, as $B\vec{y}$, defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}^5$ then multiplication by a matrix AB, as $AB\vec{z}$ defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^5$.

(A - S - N) If multiplication by a matrix A, as $A\vec{x}$, defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ and multiplication by a matrix B, as $B\vec{y}$, defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}^5$ then multiplication by a matrix BA, as $BA\vec{z}$ defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^5$.

(A - S - N) If multiplication by a matrix A, as $A\vec{x}$, defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^3$ then multiplication by AA^T defines a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$.