S09 MTH 371 Linear Algebra **QUIZ 6** Name: ________

Prof. G.Todorov March 18, 2009

1. Consider the system $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

   (a) Find the least squares solution $\vec{x}^*$.

   (b) Find $A\vec{x}^*$.

   (c) Find the error which occurs by using the least squares solution.

   (d) Find the matrix of orthogonal projection $Proj_{ImA}$ onto the subspace ImA.

   (e) Find $Proj_{ImA}\vec{b}$ onto the subspace ImA.

   (f) Find the matrix B such that $AB = Proj_{ImA}$ and $BA = I_2$, the 2×2 identity matrix.

   (g) Find $B\vec{b}$.

   (h) Sketch: $Im(A)$, $\vec{b}$, $A\vec{x}^*$, $Proj_{ImA}\vec{b}$

2. Consider the points: (0,1), (1,3), (2,0).

   (a) Sketch these points.

   (b) Find the best fitting linear model $f(t) = c_0 + c_1 t$ through these points using least squares.

   (c) Sketch the line that you obtained as the best fitting linear model on the above figure.

3. Let A be a 8×3 matrix with $rankA = 2$. Consider the linear transformation defined by multiplication by A as $A\vec{x}$. Find the following

   (a) $dimIm(A)$

   (b) $dimKer(A)$

   (c) $dim(Im(A))^{\perp}$

   (d) $dim(Ker(A))^{\perp}$

   (e) $dimIm(A^T)$

   (f) $dimKer(A^T)$

   (g) $dim(Im(A^T))^{\perp}$

   (h) $dim(Ker(A^T))^{\perp}$

   (i) $rank(A^TA)$

   (j) $rank(AA^T)$

   (k) $dimIm(A^TA)$

   (l) $dimKer(A^TA)$