S09 MTH 371 Linear Algebra **NEW PRACTICE QUIZ 6** Name: _____
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1. Consider the system $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

   (a) Sketch the $Im(A)$ and $\vec{b}$.

   (b) Find the least squares solution $\vec{x}^*$.

   (c) Find $A\vec{x}^*$.

   (d) Sketch $A\vec{x}^*$ (above).

   (e) Find $\vec{b} - A\vec{x}^*$.

   (f) Check that $(\vec{b} - A\vec{x}^*) \perp Im(A)$.

   (g) Sketch $\vec{b} - A\vec{x}^*$ (above).

   (h) Find $|\vec{b} - A\vec{x}^*|$, the length of $\vec{b} - A\vec{x}^*$.

   (i) Find the error which occurs by using the least squares solution.

   (j) Show the error on your sketch.(above)

2. Consider the points: (0,1), (0,0), (2,1).

   (a) Sketch these points.

   (b) Find the best fitting linear model $f(t) = c_0 + c_1 t$ through these points using least squares.

   (c) Sketch the line that you obtained as the best fitting linear model on the above figure.

3. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & -1 \\ 0 & 2 \end{bmatrix}$ and let $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

   (a) Find the least squares solution $\vec{x}^*$ to the equation $A\vec{x} = \vec{b}$.

   (b) Find the matrix of orthogonal projection $Proj_{ImA}$ onto the subspace ImA.

   (c) Find the matrix B such that $AB = Proj_{ImA}$ and $BA = I_2$, the 2×2 identity matrix.

4. Consider the linear transformation $\mathbb{R}^3 \to \mathbb{R}^2$ given by multiplication by the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 0 & 10 \end{bmatrix}$.

   (a) Find $Im(A)$

   (b) Sketch $Im(A)$ and $(Im(A))^{\perp}$

   (c) Find $Ker(A^T)$ and sketch on the same figure.

   (d) Find $Im(A^T)$

   (e) Sketch $Im(A^T)$ and $(Im(A^T))^{\perp}$

   (f) Find $Ker(A)$ and sketch on one of the above figures.

5. Let A be a 3×7 matrix with $rankA = 2$. Consider the linear transformation defined by multiplication by A as $A\vec{x}$. Find the following

   (a) $dimIm(A)$

   (b) $dimKer(A)$

   (c) $dim(Im(A))^{\perp}$

   (d) $dim(Ker(A))^{\perp}$

   (e) $dimIm(A^T)$

   (f) $dimKer(A^T)$

   (g) $dim(Im(A^T))^{\perp}$

   (h) $dim(Ker(A^T))^{\perp}$

   (i) $rank(A^TA)$

   (j) $rank(AA^T)$

   (k) $dimIm(A^TA)$

   (l) $dimKer(A^TA)$

   (m) $dim(Im(A^TA))^{\perp}$

   (n) $dim(Ker(A^TA))^{\perp}$

   (o) $dimIm(AA^T)$

   (p) $dimKer(AA^T)$

   (q) $dim(Im(AA^T))^{\perp}$

   (r) $dim(Ker(AA^T))^{\perp}$