

T - F If A is a 5×5 matrix, then the sum of algebraic multiplicities must be 5.

1. Find the determinants of the following matrices:

$$A = \begin{bmatrix} 9 & 0 & 5 & 0 & 6 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 0 & 3 \\ 8 & 0 & 6 & 0 & 0 & 0 & 4 \\ 4 & 0 & 3 & 0 & 0 & 0 & 2 \\ 8 & 0 & 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 7 & 0 & 0 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 0 & 0 & \dots & 0 & 0 & 8 \\ 0 & -2 & 1 & 0 & \dots & 0 & 0 & 8 \\ 0 & 0 & -3 & 1 & \dots & 0 & 0 & 8 \\ 0 & 0 & 0 & -4 & \dots & 0 & 0 & 8 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -(n-2) & 1 & 8 \\ 0 & 0 & 0 & 0 & \dots & 0 & -(n-1) & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -n \end{bmatrix}, \quad (n \times n)\text{-matrix.}$$

T - F For each eigenvalue, there is at least one eigenvector with that eigenvalue.

2. Let A be a 3×3 matrix with $\det A = 5$.

- (a) $\det(10A)$
(b) $\det(A^{-1})$
(c) $\det(AA^T)$

T - F If A is a 5×5 matrix, then for each eigenvalue there are infinitely many eigenvectors with that eigenvalue.

3. Find a matrix A which has $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ as an eigenvector with eigenvalue $\lambda_1 = 8$ and $\vec{v}_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ as an eigenvector with eigenvalue $\lambda_2 = -1$.

4. Give an example of a 3×3 matrix with exactly two different eigenvalues.

T - F A 5×5 matrix has at least one eigenvalue.

5. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

- (a) Find the characteristic polynomial of A .
- (b) Find the eigenvalues of A .
- (c) Find the algebraic multiplicities of each of the eigenvalues.
- (d) Find the eigenspaces for each of the eigenvalues.

(e) Find geometric multiplicities of each of the eigenvalues.

(f) Is there an eigenbasis? If there is, write it down; if not, explain why not.

6. Give an example of a 3×3 matrix with one eigenvalue of algebraic multiplicity 3 and geometric multiplicity 1.

7. Diagonalize the matrix $A = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}$.

T - F Geometric multiplicity of any eigenvalue is ≥ 1 .

8. Give an example of a 4×4 matrix with no eigenvalues.