S09 MTH 371 Linear Algebra **NEW PRACTICE QUIZ 7(b)** Name: ________

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1. For each of the following, either give an example (or several if asked), or explain why it does not exist. All matrices on this quiz are with entries in$\mathbb{R}$.

   (a) 2×2 matrix with two different eigenvalues.

   (b) 2×2 matrix (which is not diagonal) with two different eigenvalues.

   (c) 2×2 matrix with eigenvalues 4 and 5.

   (d) 2×2 matrix with one eigenvalue of algebraic multiplicity 2.

   (e) 2×2 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 3.

   (f) 2×2 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 2.

   (g) 2×2 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 1.

   (h) 2×2 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 0.

   (i) 2×2 matrix with no eigenvalues.

   (j) 3×3 matrix with three different eigenvalues.

   (k) 3×3 matrix with exactly two different eigenvalues.

   (l) 3×3 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 3.

   (m) 3×3 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 2.

   (n) 3×3 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 1.

   (o) 3×3 matrix with one eigenvalue of algebraic multiplicity 2 and geometric multiplicity 0.

   (p) 3×3 matrix with $\lambda_1 = 5$ an eigenvalue with algebraic and geometric multiplicity 2 and $\lambda_2 = 4$ an eigenvalue with algebraic and geometric multiplicity 1.

   (q) 3×3 matrix with $\lambda_1 = 6$ an eigenvalue with algebraic multiplicity 2 and geometric multiplicity 1 and $\lambda_2 = 7$ an eigenvalue with algebraic and geometric multiplicity 1.

   (r) 3×3 matrix with $\lambda_1 = 8$ an eigenvalue with algebraic and geometric multiplicity 2 and $\lambda_2 = -3$ an eigenvalue with algebraic multiplicity 1 and geometric multiplicity 2.

   (s) 3×3 matrix having exactly one eigenvalue with algebraic multiplicity 1 (and no other eigenvalues).

   (t) 3×3 matrix with no eigenvalues.

   (u) 4×4 matrix with no eigenvalues.

   (v) 5×5 matrix with no eigenvalues.

   (w) 3×3 matrix with 6 eigenvalues.

2. True-False (circle the correct answer)

T - F Characteristic polynomial of a 5×5 matrix has degree 5.

T - F A 5×5 matrix has at least one eigenvalue.

T - F A 5×5 matrix must have 5 different eigenvalues.

T - F Algebraic multiplicity of any eigenvalue is ≥ 1.

T - F Geometric multiplicity of any eigenvalue is ≥ 1.

T - F For any eigenvalue, the algebraic multiplicity is $\geq$ geometric multiplicity.

T - F If A is a 5×5 matrix, then the sum of algebraic multiplicities must be 5.

T - F If A is a 5×5 matrix, then the sum of algebraic multiplicities might be 5.

T - F If A is a 5×5 matrix, then the sum of geometric multiplicities must be 5.

T - F If A is a 5×5 matrix, then the sum of geometric multiplicities might be 5.

T - F Geometric multiplicity of an eigenvalue is equal to the dimension of the corresponding eigenspace.

T - F For each eigenvalue, there is at least one eigenvector with that eigenvalue.

T - F If A is a 5×5 matrix, then for each eigenvalue there are infinitely many eigenvectors with that eigenvalue.

T - F If A is a 5×5 matrix, then for each eigenvalue, there are always 5 linearly independent eigenvectors with that eigenvalue.

T - F If A is a 5×5 matrix, then for each eigenvalue, there are at most 5 linearly independent eigenvectors with that eigenvalue.

3. Diagonalize the following matrices if possible. If it is impossible, explain why.

(a) $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$