

1. Find the determinants of the following matrices:

$$(a) A = \begin{bmatrix} 1 & 0 & 4 \\ 1 & 0 & 5 \\ 1 & 2 & 6 \end{bmatrix}$$

$$(b) B = \begin{bmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{bmatrix}$$

$$(c) C = \begin{bmatrix} 9 & 0 & 5 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 4 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 0 & 3 \\ 8 & 0 & 6 & 0 & 0 & 0 & 4 \\ 8 & 0 & 6 & 0 & 3 & 0 & 5 \\ 8 & 0 & 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}$$

$$(d) D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

$$(e) E = \begin{bmatrix} 9 & 0 & 5 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 0 & 3 \\ 8 & 0 & 6 & 0 & 0 & 0 & 4 \\ 8 & 0 & 6 & 0 & 3 & 0 & 5 \\ 8 & 0 & 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}$$

$$(f) F = \begin{bmatrix} 9 & 0 & 5 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 4 & 4 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 0 & 3 \\ 8 & 0 & 6 & 0 & 0 & 0 & 4 \\ 8 & 0 & 6 & 3 & 3 & 0 & 5 \\ 8 & 0 & 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}$$

$$(g) G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

$$(h) H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 3 & 3 & 3 & 3 & 3 & 3 \\ 1 & 4 & 4 & 4 & 4 & 4 & 4 \\ 1 & 5 & 5 & 5 & 5 & 5 & 5 \\ 1 & 6 & 6 & 6 & 6 & 6 & 6 \\ 1 & 7 & 7 & 7 & 7 & 7 & 7 \end{bmatrix}$$

$$(i) J = \begin{bmatrix} a & b & c \\ d & e & c \\ f & g & c \end{bmatrix}$$

$$(j) K = [a]$$

$$(k) L = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 & 0 \\ 5 & -1 & 0 & \dots & 0 & 0 \\ 5 & 5 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 5 & 5 & 5 & \dots & -1 & 0 \\ 5 & 5 & 5 & \dots & 5 & -1 \end{bmatrix}, \text{ (55} \times \text{55)-matrix.}$$

$$(l) M = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 & 0 \\ 5 & -2 & 0 & \dots & 0 & 0 \\ 5 & 5 & -3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 5 & 5 & 5 & \dots & -(n-1) & 0 \\ 5 & 5 & 5 & \dots & 5 & -n \end{bmatrix}, \text{ (} n \times n \text{)-matrix.}$$

$$(m) N = \begin{bmatrix} (t-1) & 2t \\ (t+3) & (t+1) \end{bmatrix}$$

2. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ be (3×3) matrix such that $\det A = 5$. Find the following determinants:

(a) $\det \begin{bmatrix} 3a & 3b & 3c \\ d & e & f \\ g & h & i \end{bmatrix}$

(b) $\det \begin{bmatrix} 3a & 3b & 3c \\ 3d & 3e & 3f \\ 3g & 3h & 3i \end{bmatrix}$

(c) $\det \begin{bmatrix} a & b & c \\ d+2a & e+2b & f+2c \\ g & h & i \end{bmatrix}$

(d) $\det \begin{bmatrix} a-b & b & c \\ d-e & e & f \\ g-h & h & i \end{bmatrix}$

(e) $\det(10A)$

(f) $\det(A^2)$

(g) $\det(A^T)$

(h) $\det(A^{-1})$

(i) $\det(AA^T)$

(j) $\det((A^T A)^{-1})$

3. Consider a 3×3 matrix A with rows $\vec{v}_1, \vec{v}_2, \vec{v}_3$. Suppose $\det A = 7$. Compute:

(a) $\det \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 - 3\vec{v}_1 \\ \vec{v}_3 + \vec{v}_1 \end{bmatrix}$

(b) $\det \begin{bmatrix} \vec{v}_3 \\ \vec{v}_2 \\ \vec{v}_1 \end{bmatrix}$

(c) $\det \begin{bmatrix} 3\vec{v}_1 \\ \vec{v}_3 + \vec{v}_1 \\ \vec{v}_2 \end{bmatrix}$

4. Consider the matrix $A = \begin{bmatrix} 2 & 2 \\ 9 & 5 \end{bmatrix}$.
- Is the vector $\vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
 - Is the vector $\vec{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
 - Is the vector $\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
5. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.
- Is the vector $\vec{v} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
 - Is the vector $\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
 - Is the vector $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
 - Is the vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ an eigenvector for the matrix A . If yes, what is its eigenvalue?
6. Find a 3×3 matrix A which has \vec{e}_1 as eigenvector with eigenvalue $\lambda_1 = -2$, \vec{e}_2 as eigenvector with eigenvalue $\lambda_2 = 0$, \vec{e}_3 as eigenvector with eigenvalue $\lambda_3 = 10$.
7. Find a matrix A which has $\vec{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ as an eigenvector with eigenvalue $\lambda_1 = 5$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ as an eigenvector with eigenvalue $\lambda_2 = 3$.
8. Find all 3×3 matrices A which have \vec{e}_1 as eigenvector with eigenvalue $\lambda_1 = -2$, \vec{e}_2 as eigenvector with eigenvalue $\lambda_2 = 0$.
9. **!!!! All the HW problems in 7.2, 7.3 !!!!!**
- Find characteristic polynomial**
 - Find eigenvalues**
 - Find algebraic multiplicities of each of the eigenvalues**
 - Find eigenspaces for each of the eigenvalues**
 - Find geometric multiplicities of each of the eigenvalues**