S09 MTH 576 Rings Fields QUIZ 1 Name: ____________________

Prof. G.Todorov

Please, justify your answers.

1. Let $R = \mathbb{Z}_{12}$ (the ring of integers modulo 12).

(a) List all the elements of R and their additive inverses.

(b) List all the elements of R and their multiplicative inverses.

(c) List all zero divisors in R.

(d) Give two examples of subrings S_1 and S_2 of R.

(e) Give two examples of subsets T_1 and T_2 of R, which are NOT subrings.

(2) Describe a ring isomorphism $\mathbb{Z}_{10} \xrightarrow{f} \mathbb{Z}_2 \times \mathbb{Z}_5$.

(3) Consider the ring $R = \mathcal{M}_3(\mathbb{R})$ of all 3×3 matrices with entries in $\mathbb{R}$. Let $S \subset R$ be the following

$$S = \left\{ \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix} \mid a, b, c, d, e, f \in \mathbb{R} \right\}. \text{ Is } S \text{ a subring of } R?$$

(4) Make sure that you justify your answers!

True - False 3 is an idempotent in $\mathbb{Z}_6$.

True - False 3 is an idempotent in $\mathbb{Z}_5$.

True - False 3 is multiplicatively invertible in $\mathbb{Z}_6$.

True - False 3 is multiplicatively invertible in $\mathbb{Z}_5$.

True - False 3 is a zero divisor in $\mathbb{Z}_6$.

True - False 3 is a zero divisor in $\mathbb{Z}_5$.

(5) Give an example of a map $\mathbb{Z} \xrightarrow{f} \mathbb{Z}$ which is one-to-one and onto, but is NOT a ring isomorphism.