

Please, justify your answers. Always state clearly what statements you are using.

1. Prove that $x^2 + x + 2$ is irreducible in $\mathbb{Z}_5[x]$.

2. Prove that $x^3 - x + 2$ is irreducible in $\mathbb{Q}[x]$.

3. Prove that $x^4 + x + 1$ is irreducible in $\mathbb{Z}_2[x]$.

4. Consider the polynomial $f(x) = 2x^4 - 5x^3 + 5x - 2$
- (a) What are the rational roots of f allowed by the Rational Root Test? [List all the possibilities.]
- (b) Use the above information to factor f as a product of irreducible polynomials.
5. Give an example of a polynomial of degree 4 where Eisenstein Criterion for $p = 3$ implies that the polynomial is irreducible.
6. Give an example of a polynomial which is irreducible but where Eisenstein Criterion can not be applied.
(Justify your answer, i.e. explain why the polynomial is irreducible and why the Eisenstein Criterion does not apply.)