S09 MTH 576 Rings Fields **PRACTICE QUIZ 3** Name: ____________________

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Please, justify your answers.

1. Show that the polynomial $x^2 + 9x$ can be factored in (at least) two different ways in $\mathbb{Z}_{10}[x]$ as the product of non-constant polynomials that are not units.

2. List all monic, irreducible polynomials of degree 2 in $\mathbb{Z}_3[x]$.

3. For what values of k is $x + 1$ a factor of $x^4 + 3x^3 + 2x^2 + 2kx + 3$ in $\mathbb{Z}_5[x]$?

4. Consider the polynomial $f = x^3 + x^2 + 2x + 1$, viewed as a polynomial in $\mathbb{Z}_p[x]$. Determine whether f is irreducible when
(a) $p = 3$
(b) $p = 5$

5. Consider the polynomial $f(x) = 4x^4 + 8x^3 - 7x^2 - 11x + 6$
(a) What are the rational roots of f allowed by the Rational Root Test? [List all the possibilities.]

(b) Use the above information to factor f as a product of irreducible polynomials.

6. Show that the following polynomial $f(x)$ is irreducible in $\mathbb{Q}[x]$, by finding a prime p so that $f(x)$ is irreducible in $\mathbb{Z}_p[x]$. $f(x) = x^4 + 4x^3 + 8x^2 + 3x + 5$

7. Use Eisensteins Criterion to show that the following polynomial is irreducible in $\mathbb{Q}[x]$. [Indicate which prime is used, and how the Criterion applies. You will need to perform a preliminary change of variable, of the form $x \to x + c$, for some suitable constant c.] $f(x) = x^4 - x^3 + x^2 - x + 1$

8. "True - False" for polynomials in $F[x]$, where F is a field:

T - F If $f(x)$ is irreducible in $F(x)$, then $f(x)$ has no roots in F.

T - F Every polynomial in $F[x]$ of degree 2 has a root in F.

T - F A polynomial of degree 3 in $F(x)$ is irreducible if and only if it has no roots.

T - F An element $a \in F$ is a root of the polynomial $f(x) \in F(x)$ if and only if $(x - a)$ divides $f(x)$.

9. Give examples of:

(a) A polynomial $f(x) \in \mathbb{Z}[x]$ for which Eisenstein criterion for $p = 2$ implies that the polynomial is irreducible.

(b) A polynomial $f(x) \in \mathbb{Z}[x]$ for which Eisenstein criterion for $p = 2$ does not apply, but for $p = 3$ implies that the polynomial is irreducible.

(c) A monic polynomial $f(x) \in \mathbb{Z}[x]$ with all even non-leading coefficients for which Eisenstein criterion for $p = 2$ does not apply.

(d) A monic polynomial $f(x) \in \mathbb{Z}[x]$ with all even non-leading coefficients which is reducible.