

1. Consider the congruence-class ring  $S = \mathbb{Z}_3[x]/(x^2 + 2)$ .
  - (a) How many congruence classes modulo  $x^2 + 2$  are there in  $\mathbb{Z}_3[x]$ ? List them all.
  - (b) Find the congruence class of  $[2x^4 + x + 2]$ .
  - (c) Write the general formula for multiplication in  $S$ .
  - (d) Which elements in  $\mathbb{Z}_3[x]/(x^2 + 2)$  have multiplicative inverses?
  - (e) Write the inverses for the elements which have inverses.
  - (f) Which elements in  $\mathbb{Z}_3[x]/(x^2 + 2)$  do NOT have multiplicative inverses?
  - (g) Which elements are zero divisors in  $\mathbb{Z}_3[x]/(x^2 + 2)$ ? Explain.
  - (h) Is the ring  $\mathbb{Z}_3[x]/(x^2 + 2)$  a field? Explain.

2. Which of the following congruence-class rings is a field? Explain.

(a)  $\mathbb{Q}[x]/(x^3 - 3x^2 + 3x - 3)$

(b)  $\mathbb{Q}[x]/(x^3 - 3x^2 + x - 3)$

3. Let  $[f(x)] = [x^2 + x + 1] \in \mathbb{Z}_2[x]/p(x)$  where  $p(x) = (x^4 + 1)$ .

(a) Show that  $\gcd(f(x), p(x)) = 1$ .

(b) Express the  $\gcd(f(x), p(x))$  as  $\gcd(f(x), p(x)) = u(x)f(x) + v(x)p(x)$ .

(c) Find  $[f(x)]^{-1}$ .