S09 MTH 576 Rings Fields **PRACTICE QUIZ 4** Name: ____________________

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Please, justify your answers.

You may use the theorem that $F[x]/p(x)$ is a field if and only if $p(x)$ is irreducible in $F[x]$.

1. How many congruence classes modulo $x^2 + x + 2$ are there in $\mathbb{Z}_3[x]$? List them all.

2. Consider the congruence-class ring $S = \mathbb{Z}_3[x]/(x^2 + x + 2)$.
   (a) Find the congruence class of $[x^3]$ using the list from the above problem.

   (b) Find the congruence class of $[2x^4 + x + 2]$ using the list from the above problem.

3. Consider the congruence-class ring $S = \mathbb{Z}_2[x]/(x^2 + x)$.
   (a) Write out the addition and multiplication tables for S .

   (b) What are the units (if any) in S ?

   (c) What are the zero-divisors (if any) in S?

4. Which of the following congruence-class rings is a field? Explain.
   (a) $\mathbb{Q}[x]/(x^3 - 2x^2 + 2x - 2)$

   (b) $\mathbb{Q}[x]/(x^3 - 2x^2 + x - 2)$

5. Consider the congruence-class ring $S = \mathbb{Z}_3[x]/(x^2 + x)$.

   (a) How many congruence classes are there? List them all.

   (b) Write the general formula for multiplication in S.

   (c) Which elements in $\mathbb{Z}_3[x]/(x^2 + x)$ have multiplicative inverses?

   (d) Write the inverses for the elements which have inverses.

   (e) Which elements in $\mathbb{Z}_3[x]/(x^2 + x)$ do NOT have multiplicative inverses? Why?

   (f) Which elements are zero divisors in $\mathbb{Z}_3[x]/(x^2 + x)$?

   (g) Write out the multiplication table for S .

   (h) What are the units (if any) in S (using the table)?

   (i) What are the zero-divisors (if any) in S (using the table)?

   (j) Is the ring $\mathbb{Z}_3[x]/(x^2 + x)$ a field? Explain.

   (k) Give an independent alternative proof using the theorem (of whatever your previous answer is).

6. Consider the congruence-class ring $S = \mathbb{Q}[x]/(x^2 - 5)$.

(a) How many congruence classes are there? Describe them.

(b) Write the general formula for multiplication in S.

(c) Which elements in $\mathbb{Q}[x]/(x^2 - 5)$ have multiplicative inverses?

(d) Write the inverses for the elements which have inverses.

(e) Which elements in $\mathbb{Q}[x]/(x^2 - 5)$ do NOT have multiplicative inverses? Why?

(f) Which elements are zero divisors in $\mathbb{Q}[x]/(x^2 - 5)$?

(g) Is the ring $\mathbb{Q}[x]/(x^2 - 5)$ a field? Explain.

(h) Give an independent alternative proof using the theorem (of whatever your previous answer is).

7. Consider $\mathbb{Z}_3[x]/(x^2+x+2)$. Find the multiplicative inverse of $[2x^4+x+2]$.

8. Explain why is $[f(x)] = [x^2+x+1] \in \mathbb{Z}_2[x]/(x^3+x+1)$ a unit and find its inverse.

   (a) Let $f(x) = x^2+x+1, p(x) = (x^3+x+1)$. Prove that $f(x)$ and $p(x)$ are relatively prime.

   (b) Show that $gcd(f(x), p(x)) = 1$ by actually finding the gcd.

   (c) Express the gcd, i.e. $1 = u(x)f(x) + v(x)p(x)$

   (d) Explain why is $[f(x)]^{-1} = [u(x)]$.