

1. Consider the polynomial  $p(x) = x^2 + 2x + 4 \in \mathbb{Z}_5[x]$ . Let  $K = \mathbb{Z}_5[x]/(x^2 + 2x + 4)$  be the congruence-class ring.

(a) Prove that  $K$  is a field.

(b) How many elements are there in  $K$ ?

(c) Prove that the element  $[4x + 3] \in K$  is a root of the polynomial  $p(x) \in K[x]$ .

(d) Factor the polynomial  $p(x) = (x - a)(x - b) \in K[x]$ , i.e. with  $a, b \in K$ . (Please explain.)

2. Give an example of a ring  $R$  without identity  $1_R$ . (Please explain.)

3. Give an example of a ring and a subring which is not an ideal. (Please explain.)

4. Give an example of a field with 8 elements. Write the multiplication table for this field.

5. Consider the map  $f : \mathbb{Z}_{24} \rightarrow \mathbb{Z}_8$  defined as  $f([n]_{24}) := [n]_8$ , the congruence classes of  $n$  mod 24 and 8 (respectively).

(a) Prove that  $f$  is well defined.

(b) Prove that  $f$  is ring homomorphism.

(c) What is  $Im(f)$ ?

(d) What is  $Ker(f)$ ?