SU109 MTH 371 Linear Algebra

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PRACTICE QUIZ 1

Name: ____________________

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Please, justify your answers.

1. Consider the following system of equations:

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & - & x_3 & + & 3x_4 & = & 0 \\ -2x_1 & + & 4x_2 & + & 5x_3 & - & 5x_4 & = & 3 \\ 3x_1 & - & 6x_2 & - & 6x_3 & + & 8x_4 & = & -3 \end{array}$$

   (a) Write the matrix of coefficients A for this system.

   (b) Write the augmented B for this system.

   (c) Find the $rref B$. Indicate for each step which row operation you use.

   (d) Which are the leading variables?

   (e) Which are the non-leading (free) variables?

   (f) Write down the general solution of the system, in vector form.

2. Consider the row reduced echelon forms of the augmented matrices $B = [\ A \vdots \vec{b}\ ]$ of 3 systems of equations:

   (a) $\begin{bmatrix} 1 & 2 & 0 & \vdots\, 0 \\ 0 & 0 & 1 & \vdots\, 0 \\ 0 & 0 & 0 & \vdots\, 1 \end{bmatrix}$

   Find rank$A$:

   Find rank$B$:

   How many solutions does this system have:

   (b) $\begin{bmatrix} 0 & 0 & 1 & \vdots\, 2 \\ 0 & 0 & 0 & \vdots\, 0 \\ 0 & 0 & 0 & \vdots\, 0 \end{bmatrix}$

   Find rank$A$:

   Find rank$B$:

   How many solutions does this system have:

   (c) $\begin{bmatrix} 1 & 0 & 0 & \vdots\, 0 \\ 0 & 1 & 0 & \vdots\, 1 \\ 0 & 0 & 1 & \vdots\, 2 \end{bmatrix}$

   Find rank$A$:

   Find rank$B$:

   How many solutions does this system have :

3. For a linear system $A\vec{x} = \vec{b}$, you are given either the rank of the coefficient matrix A, or the rank of the augmented matrix $B = [\, A \vdots \vec{b}\,]$. In each case indicate what are the possible values for the missing rank (all possible), and in each possible case state whether the system could have no solution, one solution, or infinitely many solutions.

$\#\{equations\}$	$\#\{variables\}$	$rankA$	$rk[\, A \vdots \vec{b}\,]$	$\#\{solutions\}$
5	4	4		
4	5		3	
5	4		5	
4	5	4		

4. Decide if the following matrices are in rref.

$$\begin{bmatrix}1\\1\end{bmatrix}, \quad \begin{bmatrix}1 & 1\end{bmatrix}, \quad \begin{bmatrix}1\\0\end{bmatrix}, \quad \begin{bmatrix}1 & 0\end{bmatrix}, \quad \begin{bmatrix}0\\1\end{bmatrix}, \quad \begin{bmatrix}0 & 1\end{bmatrix},$$

$$\begin{bmatrix}1\\3\end{bmatrix}, \quad \begin{bmatrix}1 & 3\end{bmatrix}, \quad \begin{bmatrix}0\\0\end{bmatrix}, \quad \begin{bmatrix}0 & 0\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \begin{bmatrix}0 & 1 & 0\end{bmatrix},$$

$$\begin{bmatrix}1 & 1\\3 & 0\end{bmatrix}, \quad \begin{bmatrix}1 & 3\\0 & 0\end{bmatrix}, \quad \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}, \quad \begin{bmatrix}1 & 3\\0 & 0\\0 & 1\end{bmatrix}, \quad \begin{bmatrix}1 & 3 & 4 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}, \quad \begin{bmatrix}1 & 3 & 4 & 1\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix},$$

5. EXAMPLES (some of these might not exist)

   (a) Give an example of a square matrix A with $rkA = 1$

   (b) Give an example of a square matrix A with $rkA = 3$

   (c) Give an example of a 2×5 matrix A with $rkA = 1$

   (d) Give an example of a 5×2 matrix A with $rkA = 1$

   (e) Give an example of a 2×5 matrix A with $rkA = 2$

   (f) Give an example of a 5×2 matrix A with $rkA = 2$

   (g) Give an example of a 2×5 matrix A with $rkA = 3$

(h) Give an example of a 5×2 matrix A with $rkA = 3$

(i) Give an example of a 2×5 matrix A with $rkA = 5$

(j) Give an example of a 5×2 matrix A with $rkA = 5$

6. EXAMPLES - give examples of systems of equations as stated below (some of these might not exist):

   (a) 2 equations, 3 variables
   (b) 4 equations, 3 variables
   (c) 1 equation, 3 variables
   (d) 3 equations, 1 variable
   (e) System which has exactly one solution.
   (f) System which has infinitely many solutions.
   (g) System which has no solutions.
   (h) 2 equations, 3 variables with a unique solution.
   (i) 4 equations, 3 variables with a unique solution.
   (j) 2 equations, 3 variables with infinitely many solutions.
   (k) 4 equations, 3 variables with infinitely many solutions.
   (l) 2 equations, 3 variables with no solutions.
   (m) 4 equations, 3 variables with no solutions.

7. Write down all possible types of $rref$ for

   (a) 3×3 matrices
   (b) 2×4 matrices
   (c) 4×3 matrices
   (d) 3×6 matrices of rank 1
   (e) 5×3 matrices of rank 2
   (f) 3×3 matrices of rank 0

8. Always-Sometimes-Never (circle A or S or N)

   Let A be the matrix of coefficients and $B = [\ A \vdots \vec{b}\ ]$ the augmented matrix of a system of 3 equations in 8 variables. Then:

A - S - N A is a 3×8 matrix.

A - S - N B is a 3×9 matrix.

A - S - N $rankA = 8$

A - S - N If $rankA = 3$ then $rankB = 3$.

A - S - N If $rankA = 3$ then $rankB = 4$.

A - S - N If $rankA = 3$ then $rankB = 8$.

A - S - N If $rankA = 1$ then $rankB = 2$.

A - S - N If $rankA = 1$ then $rankB \leq 2$.

A - S - N If $rankA = rankB$ then the system has a unique solution.

A - S - N If $rankA = rankB$ then this system has infinitely many solutions.

A - S - N If $rankA = rankB$ then the system has no solutions.

A - S - N If $rankA \neq rankB$ then the system has a unique solution.

A - S - N If $rankA \neq rankB$ then this system has infinitely many solutions.

A - S - N If $rankA \neq rankB$ then the system has no solutions.

9. Always-Sometimes-Never (circle A or S or N)

A - S - N If A is 3×5 matrix, then $rkA = 3$.

A - S - N If A is 3×5 matrix, then $rkA = 5$.

A - S - N If A is 3×5 matrix, then $rkA = 1$.

A - S - N If $(\vec{u} \cdot \vec{v}) = 0$ then $(\vec{u} \perp \vec{v})$

A - S - N If a system of 3 equations in 3 variables has a unique solution then the rref of the coefficient matrix is the identity matrix.

A - S - N If a system of 3 equations in 3 variables has infinitely many solutions then the rref of the coefficient matrix is the identity matrix.

A - S - N If a system of 3 equations in 3 variables has no solutions then the rref of the coefficient matrix is the identity matrix.

10. Let $\vec{u} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$.

   (a) Find $(\vec{u} \cdot \vec{v})$
   (b) Find $(\vec{u} \cdot \vec{w})$
   (c) Find $(\vec{w} \cdot \vec{y})$
   (d) Find all vectors $\vec{x}$ such that $\vec{x} \perp \vec{w}$.